Short wavelength limits for control and measurement of collective micro-dynamic noise suppression/gain

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OUTLINE

Suppression/enhancement of noise:

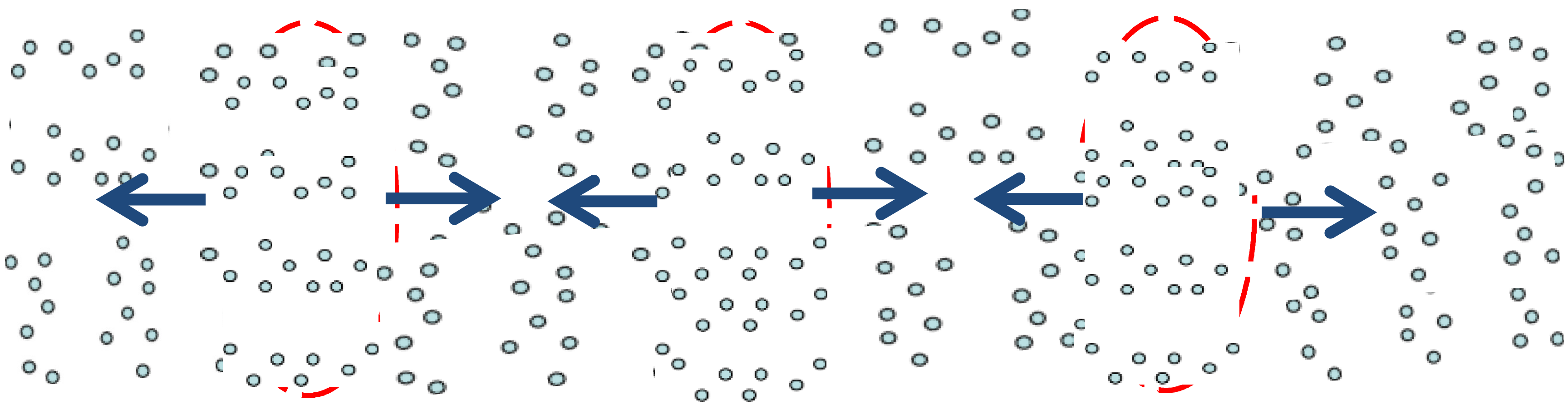
- Quarter plasma oscillation beam drift scheme.
- Beam drift/dispersion scheme
- Comparison of schemes and short wavelength limits.

Measurement of current noise

- Shot-Noise and the Sum-Rule theorem.
- Fundamental limits of noise measurement.
- Measuring current noise and radiation noise.

Collective effect (LSC):

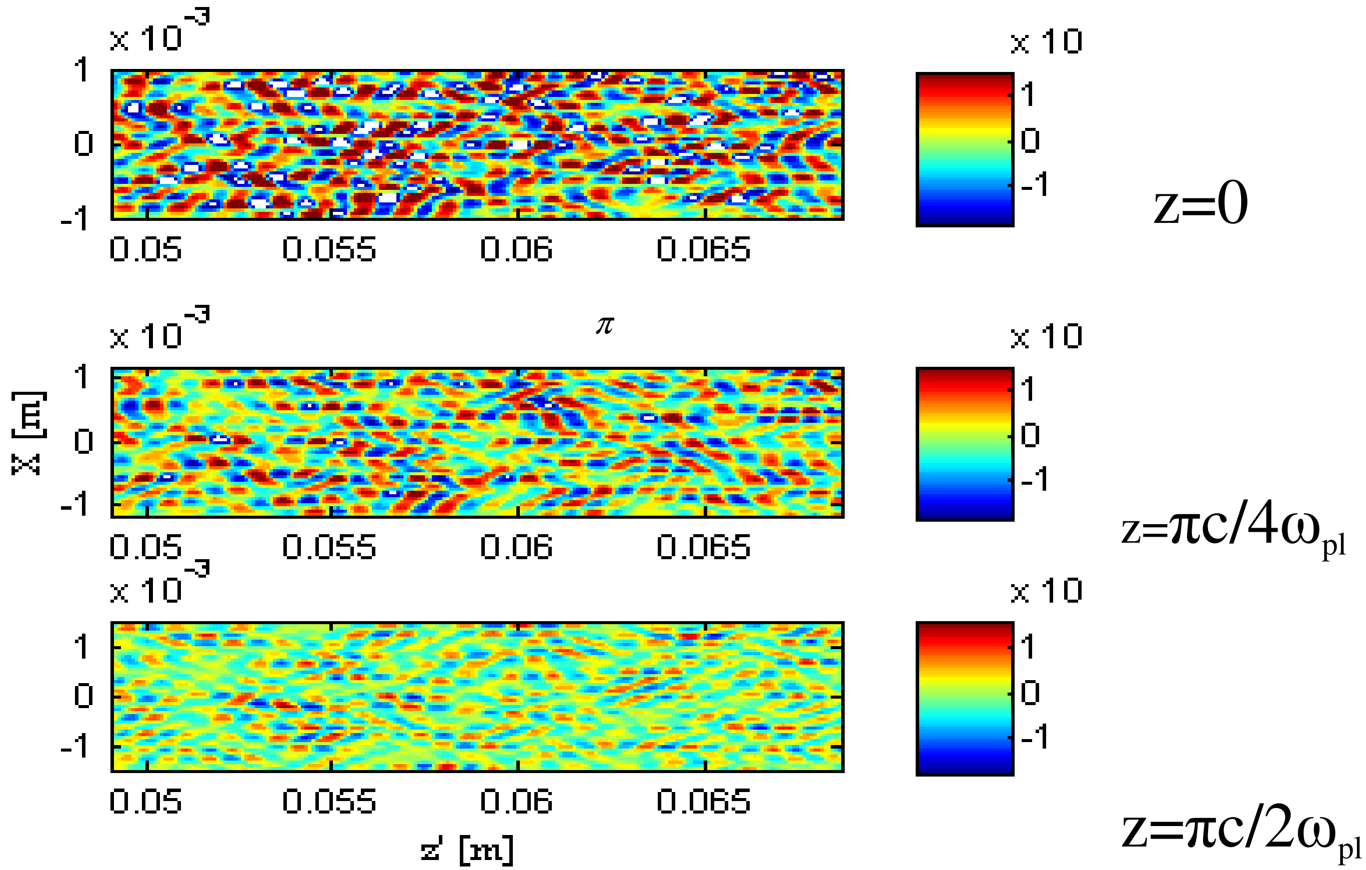
1. Space-charge expansion of random bunches.
2. Development of correlated velocity distribution.
3. Longitudinal plasma wave oscillation.
4. Effect of dispersion.



Shot-Noise spectral power: $\overline{\left|\check{I}(f)\right|^2} = eI_b \quad [A^2\text{-Sec}]$

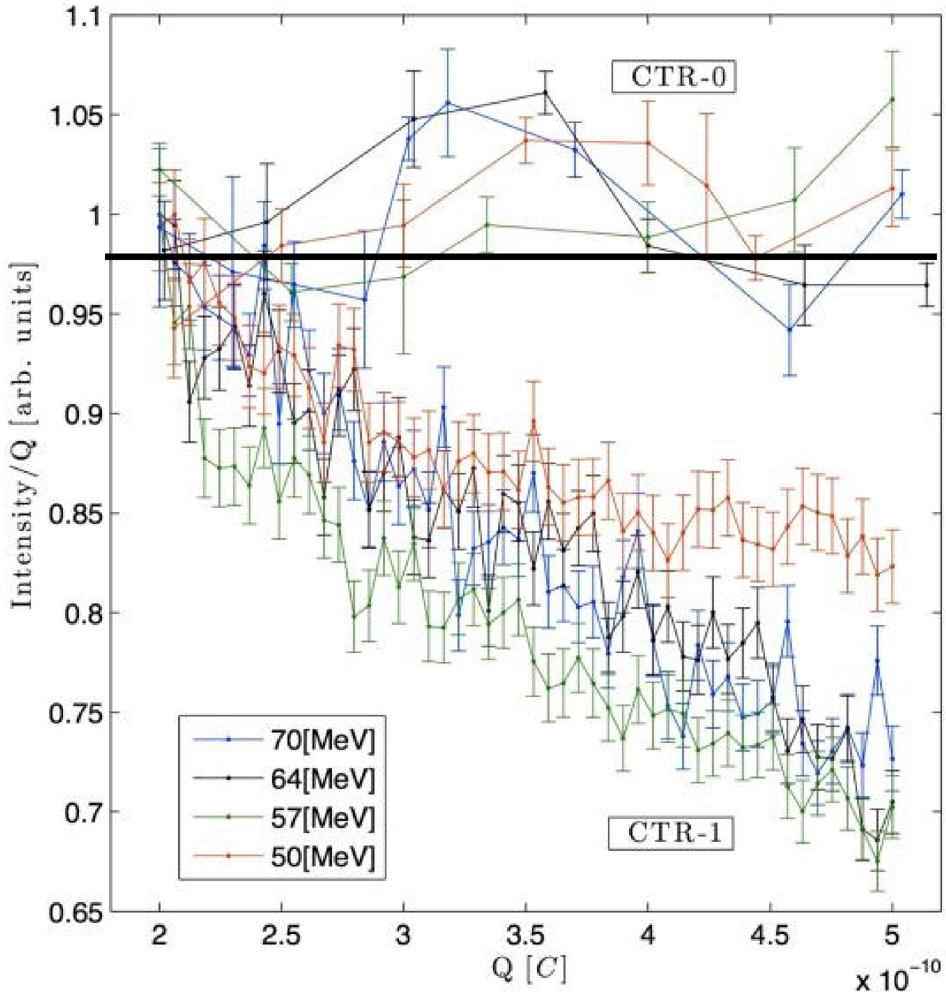
Charge Density Homogenization – Axially Filtered 5-10 [μm]

Simulation Parameters (60k macro-particles):
FERMI: E= 100 [MeV], R=1 [mm], I = 80 [A]



A. Nause, E. Dyunin, A. Gover, "Optical frequency Shot- Noise suppression in electron beams: 3-D analysis", J. of Applied Physics 107, 103101 (2010).

Measured OTR Signal per unit charge

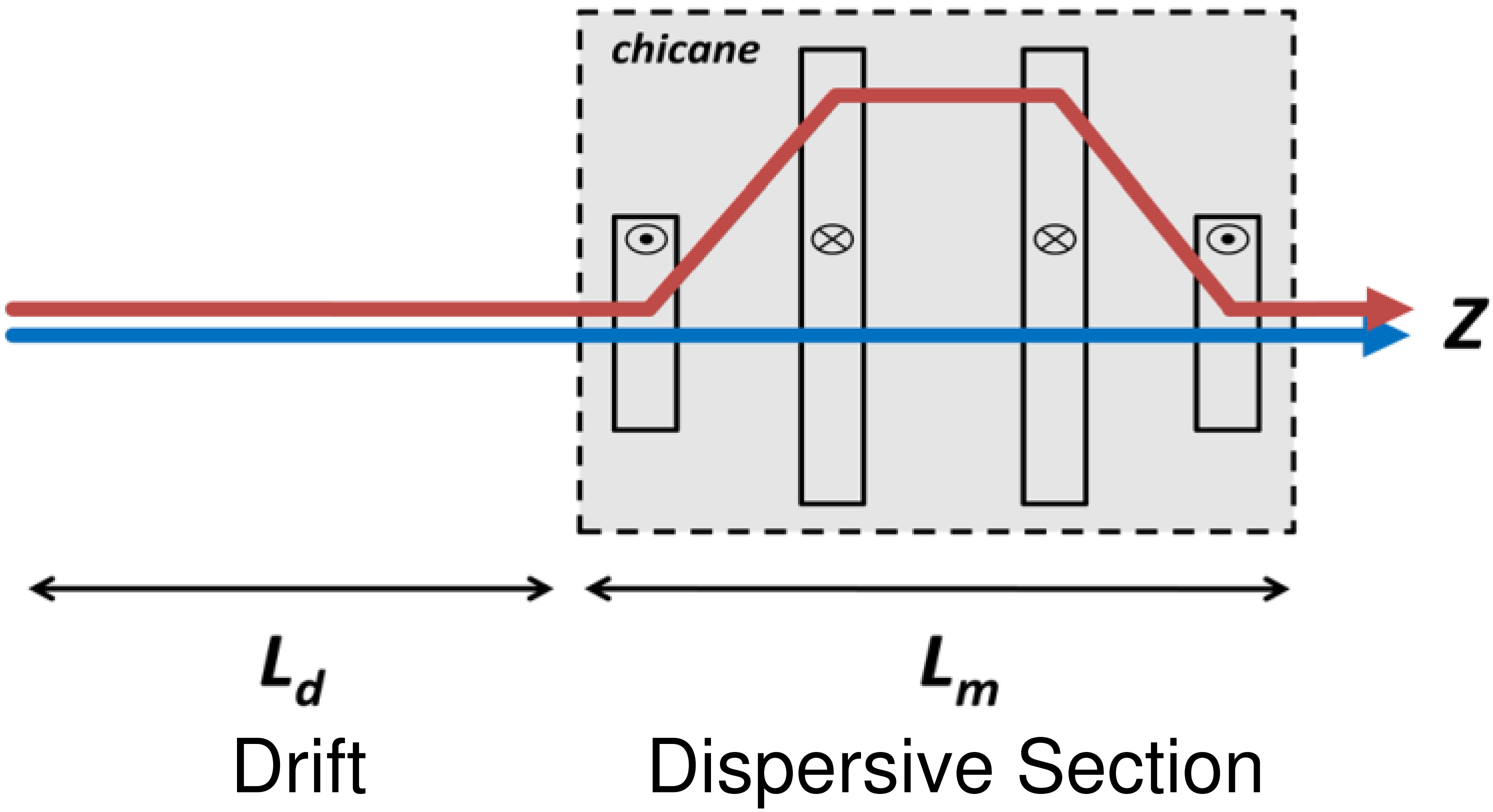


Before drift: linear dependence on Q

After drift: sub-linear dependence on Q

A. Gover, A. Nause, E. Dyunin, M. Fedurin "Beating the shot-noise limit", Nature Physics, Vol. 8, No. 12 pp. 877-880 (Dec. 2012).

Drift / Dispersion Transport



D. Ratner Z. Huang G. Stupakov, Phys. Rev. ST-AB, **14**, 060710 (2011)
A.Gover, E.Dyunin, T.Duchovni, A.Nause, *Phys. of Plasmas*, **18,** 123102 (2011).

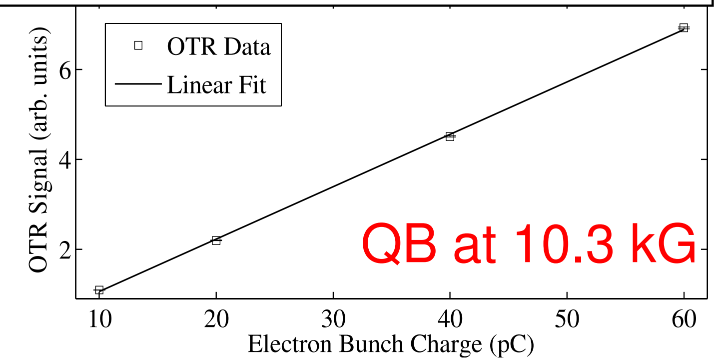
Dispersive Shot Noise Supp.

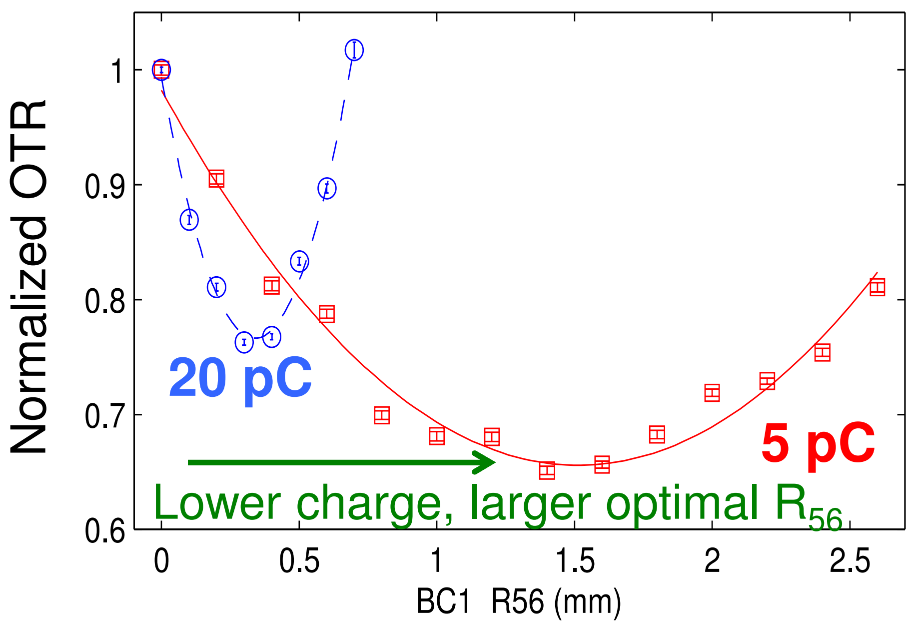
1D Dispersive Shot Noise Suppression

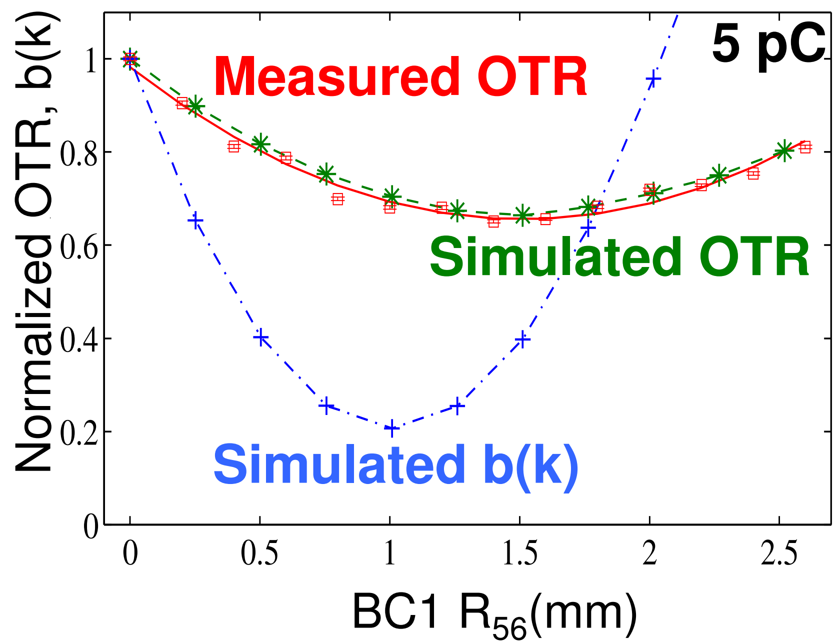
$$N\left\langle |b(k)|^2 \right\rangle = (1-\Upsilon)^2$$

$$\Upsilon \equiv n_0 R_{56} A$$

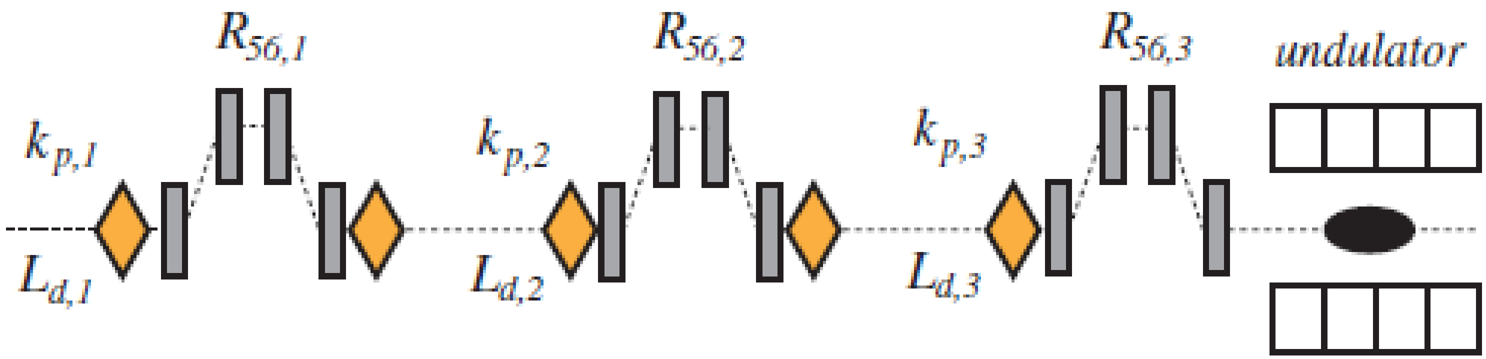
OTR proportional to charge at start







Cascaded Longitudinal Space-Charge Amplifier - NLCTA



$$\mathbf{M} = \begin{pmatrix} \cos(k_p L_d) & -\frac{ik}{\gamma^2 k_p}\sin(k_p L_d) \\ \frac{k_p \gamma^2}{ik}\sin(k_p L_d) & \cos(k_p L_d) \end{pmatrix}$$

$$\begin{pmatrix} b_{N_s} \\ \mu_{N_s} \end{pmatrix} = \mathbf{R}_{\mathbf{N_s}}\mathbf{M}_{\mathbf{N_s}} \ldots \mathbf{R}_2\mathbf{M}_2\mathbf{R}_1\mathbf{M}_1 \begin{pmatrix} b_0 \\ \mu_0 \end{pmatrix}.$$

Marinelli *et al,* PRL 110, 264802 (2013)

E. Schneidmiller, M.V. Yurkov, PRST 13, 110701 (2010).

S. Seletskiy *et al* PRL 111, 034803 (2013)

COMPREHNSIVE MODEL FOR LSC MICRODYNAMICS IN DRIFT AND DISPERSION

A. Nause, E. Dyunin, A. Gover,
“Short wavelength limits of current shot noise suppression”
PHYSICS OF PLASMAS 21, 083114 (2014)

Coherent Plasma Oscillation in a Drift Section

$$\breve{i}(L_d,\omega) = \left[\breve{i}(0,\omega)\cos\phi_p - i\breve{V}(0,\omega)\left(\sin\phi_p / W_d\right)\right] e^{i\phi_b(L_d)}$$

$$\breve{V}(L_d,\omega) = \left[-i\breve{i}(0,\omega) W_d \sin\phi_p + \breve{V}(0,\omega)\cos\phi_p\right] e^{i\phi_b(L_d)}$$

$$\breve{V}(z,\omega) = -\left(mc^2/e\right)\breve{\gamma}(z,\omega) = -\left(mc^2/e\right)\gamma_0^3 v_0 \breve{v}(\omega)$$

(Chu's Relativistic Kinetic Voltage)

$$\phi_b = \frac{\omega}{v_z} L_d \qquad \phi_p = \theta_{pr} L_d \qquad \theta_{pr} = r_p \frac{\omega_p'}{v_0}$$

$$\omega_p' = \left(\frac{e^2 n_0}{m\varepsilon_0 \gamma^3}\right)^{1/2} \qquad W_d = \sqrt{\mu_0/\varepsilon_0} \Big/ k\theta_{prd} A_e$$

[A. Gover, E. Dyunin, PRL 102, 154801 (2009)]

TRANSFER MATRIX FOR UNIFORM DRIFT LSC

$$M_d = \begin{pmatrix} \cos\phi_{pr} & -i\frac{i}{W_d}\sin\phi_{pr} \\ -iW_d\sin\phi_{pr} & \cos\phi_{pr} \end{pmatrix}$$

$$W = -iZ_{LSC}/\theta_{pr}$$

Current Shot-Noise Suppression

$$gain = \frac{\overline{\left|\breve{i}\,(L_d,\omega)\right|^2}}{\overline{\left|\breve{i}\,(0,\omega)\right|^2}} = \cos^2\phi_p + N^2\sin^2\phi_p$$

$$N^2 = \frac{\overline{\left|\breve{V}(0,\omega)\right|^2}}{W_d^2\overline{\left|\breve{i}\,(0,\omega)\right|^2}}$$

$$Gain(\varphi_p = \pi/2) = N^2$$

$\langle\langle\, 1$ **For current noise dominated beam.**

Significance of N^2

Noise dominance parameter

$$N^2 \equiv \frac{\overline{\left|\breve{v}(0,\omega)\right|^2}}{\overline{\left|\breve{i}(0,\omega)\right|^2 W_d^2}}$$

Minimal gain factor in drift

$$gain\Big|_{\phi_{bd}=\pi/2} = N^2$$

Landau-damping parameter

$$N_D = \frac{k}{k_D} \qquad \left(k_D = \frac{2\pi}{\lambda_D} = \frac{\omega_{pL}}{\delta v_z} \right)$$

$$N_d = \mathrm{N}$$

Phase-spread parameter

$$\Delta\varphi_b = kL_d \frac{\Delta\beta_z}{\beta_z^2} = kL_d \frac{\Delta\beta_z c}{\omega_{pr}\beta_z^2} \frac{\omega_{pr}}{c} = \frac{k}{k_D} \frac{L_d \theta_{pr}}{\beta_z} = N\phi_{prd}$$

$$\Delta\varphi_b\Big|_{\phi_{pd}=\pi/2} = \frac{\pi}{2} N$$

TRANSFER MATRIX FOR DISPESIVE SECTION

$$\begin{pmatrix} \breve{i}(L,\omega) \\ \breve{v}(L,\omega) \end{pmatrix} =$$

$$\begin{pmatrix} 1-\int_0^{\phi_p(L)} \frac{1}{W(\phi_p)} \int_0^{\phi_p} W(\phi_p')d\phi_p' d\phi_p & -i\int_0^{\phi_p(L)} \frac{d\phi_p}{W(\phi_p)} \\ -i\int_0^{\phi_p(L)} W(\phi_p)d\phi_p & 1-\int_0^{\phi_p(L)} W(\phi_p)\int_0^{\phi_p} \frac{d\phi_p'}{W(\phi_p')} d\phi_p \end{pmatrix}$$

$$\times \begin{pmatrix} \breve{i}(0,\omega) \\ \breve{v}(0,\omega) \end{pmatrix} \quad (38)$$

$$\underline{\underline{M}}_m = \begin{pmatrix} 1-\theta_{prd}^2 \int_0^{L_m} z(1+a_\perp^2(z))dz & -i\frac{\theta_{prd}}{W_d}\int_0^{L_m}(1+a_\perp^2(z))dz \\ -iW_d\theta_{prd}L_m & 1-\theta_{prd}^2\int_0^{L_m}\int_0^{z}(1+a_\perp^2(z))dz'dz \end{pmatrix}$$

$$\underline{\underline{M}}_m = \begin{pmatrix} 1 & i\frac{\gamma_0^2\theta_{prd}}{W_d}R_{56} \\ -iW_d\theta_{prd}L_m & 1 \end{pmatrix}$$

$$R_{56} = -\frac{1}{\gamma_0^2}\int_0^{L_m}(1+a_\perp^2(z))dz$$

Dispersive Transport Noise Suppression

$$gain = \frac{\overline{\left|\breve{i}(L,\omega)\right|^2}}{\overline{\left|\breve{i}(0,\omega)\right|^2}} = \left(\cos\phi_{pd} + \gamma_0^2\theta_{pd}R_{56}\sin\phi_{pd}\right)^2 + N^2\left(\sin\phi_{pd} - \gamma_0^2\theta_{pd}R_{56}\cos\phi_{pd}\right)^2$$

$$K_d = \frac{\gamma_0^2\left|R_{56}\right|}{L_d} \qquad N << 1 \qquad \phi_{pd} << 1$$

$$gain = \left(1 - K_d\phi_{pd}^2\right)^2 + N^2\phi_{pd}^2\left(1 + K_d\right)^2$$

[This is equivalent to Ratner *et al* for N=0 and assuming $kR_{56}\,\Delta\gamma/\gamma \ll 1$]

<u>For maximal suppression:</u> $(N^2 << \varphi_{pd} << 1)$

$$\begin{cases} (K_d)_{\min} = \dfrac{1}{\varphi_{pd}^2} \\ (gain)_{\min} = \dfrac{N^2}{\varphi_{pd}^2} \end{cases}$$

COMPARISON OF DRIFT & DRIFT/DISPERSION SCHEMES

	Drift	Drift/dispersion
Optimal drift phase ϕ_{pd}	$\pi/2$	ϕ_{pd}
Optimal dispersion $K_d = \gamma_0^2 \lvert R_{56} \rvert / L_d$	0	$1/\phi_{pd}^2$
Suppression factor G_{min}	N^2	N^2/ϕ_{pd}^2
Shortest wavelength λ_{min} (for a given G_{min})	$\lambda_p \Delta\beta_z G_{min}^{-1/2}$	$\lambda_p \Delta\beta_z G_{min}^{-1/2}/\phi_{pd}$
Shortest wavelength λ (for validity of scheme)	$\ll \frac{1}{2}\lambda_p \Delta\beta_z$	$\frac{1}{\pi}\lambda_p \Delta\beta_z/\phi_{pd}$

CURRENT-NOISE MEASUREMENT

Derivation of the Shot-Noise Formula for A Finite Bunch of Particles

Spatial (longitudinal) “Energy” Spectral Density (ESD) of zero dimension particles:

$$N(z) = \sum_{j=1}^{N} \delta\left(z - z_j\right) \qquad \breve{N}(k) = \int_{-\infty}^{\infty} e^{-ikz} N(z)dz = \sum_{j=1}^{N} e^{-ikz_j}$$

Parseval Thm:

?

$$\mathrm{E} = \int_{-\infty}^{\infty} N^2(z)dz = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\breve{N}(k)\right|^2 dk = \int_{-\infty}^{\infty} p(k)dk$$

$$p(k) = \frac{1}{2\pi}\left|\breve{N}(k)\right|^2 = \frac{1}{2\pi}\left|\sum_{i,j}^{N} e^{-ikz_j}\right|^2 = \frac{1}{2\pi}\left[N + \sum_{i\neq j}^{N} e^{-ik\left(z_i - z_j\right)}\right]$$

If z_i, z_j are uncorrelated and random (Shot-Noise):

$$p(k) = \frac{N}{2\pi} \qquad p_+(k) = 2p = \frac{N}{\pi}$$

Shot-Noise: Time Domain Description

$$I(t) = -e\sum_{j=1}^{N}\delta\left(t - t_{0j}\right) \qquad \check{I}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} I(t)dt = -e\sum_{j=1}^{N} e^{i\omega t_{0j}}$$

$$\int_{-\infty}^{\infty} I^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\check{I}(\omega)|^2 d\omega \equiv \int_{-\infty}^{\infty} p(\omega)d\omega$$

$$p(\omega) = \frac{e^2}{2\pi}\left|\sum_{j}^{N} e^{i\omega t_{0j}}\right|^2 = \frac{e^2}{2\pi}\mathrm{N}$$

Power Spectral Density (PSD): for a random coasting beam: $S_I(\omega) = p(\omega)/T$

$$S_I(\omega) = \frac{e^2 N}{2\pi T} = \frac{eI_0}{2\pi}, \qquad S_{I_+}(\omega) = 2S_I(\omega) = \frac{eI_0}{\pi}$$

($S_I(f)=2eI_0$) (White noise: infinite energy!)

MEASUREMENT OF BEAM CURRENT NOISE BY RADIATION EMISSION

Assume frozen particle distribution during measurement.

For inclusion of LSC dynamics during radiation (SASE) see:

A. Gover, E. Dyunin, “Coherence Limits of Free Electron Lasers”

IEEE J. Quant. Electron. **46**, 1511 (2010)

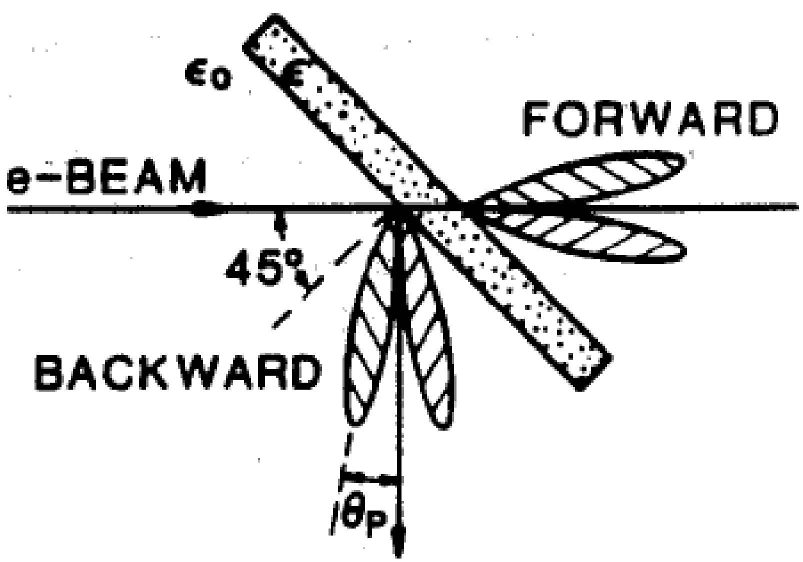
FAR FIELD MEASUREMENT

$$\frac{d^2\breve{I}}{d\Omega d\omega} = \frac{d^2 I_e}{d\Omega d\omega} N^2 |M_b(\theta_x, \theta_y, \omega)|^2$$

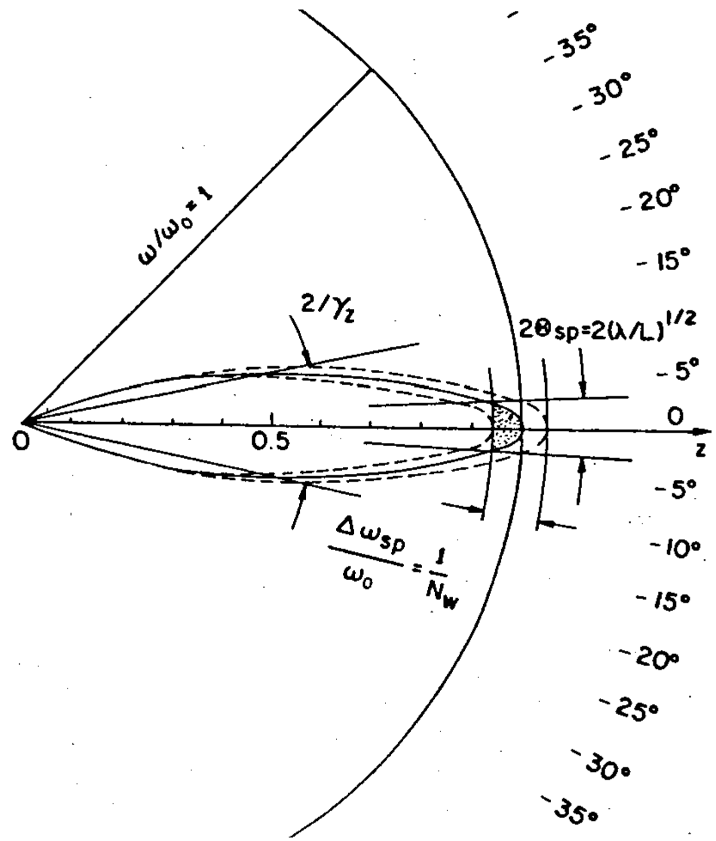
$$M_b(\theta_x, \theta_y, \omega) = \overline{\frac{1}{N}\sum_{j=1}^{N} \exp\left[-ik(\sin\theta_x x_{0j} + \sin\theta_y y_{0j} + z_{0j}/\beta)\right]}$$

SPECTRAL RADIANT INTENSITY FROM A SINGLE ELECTRON

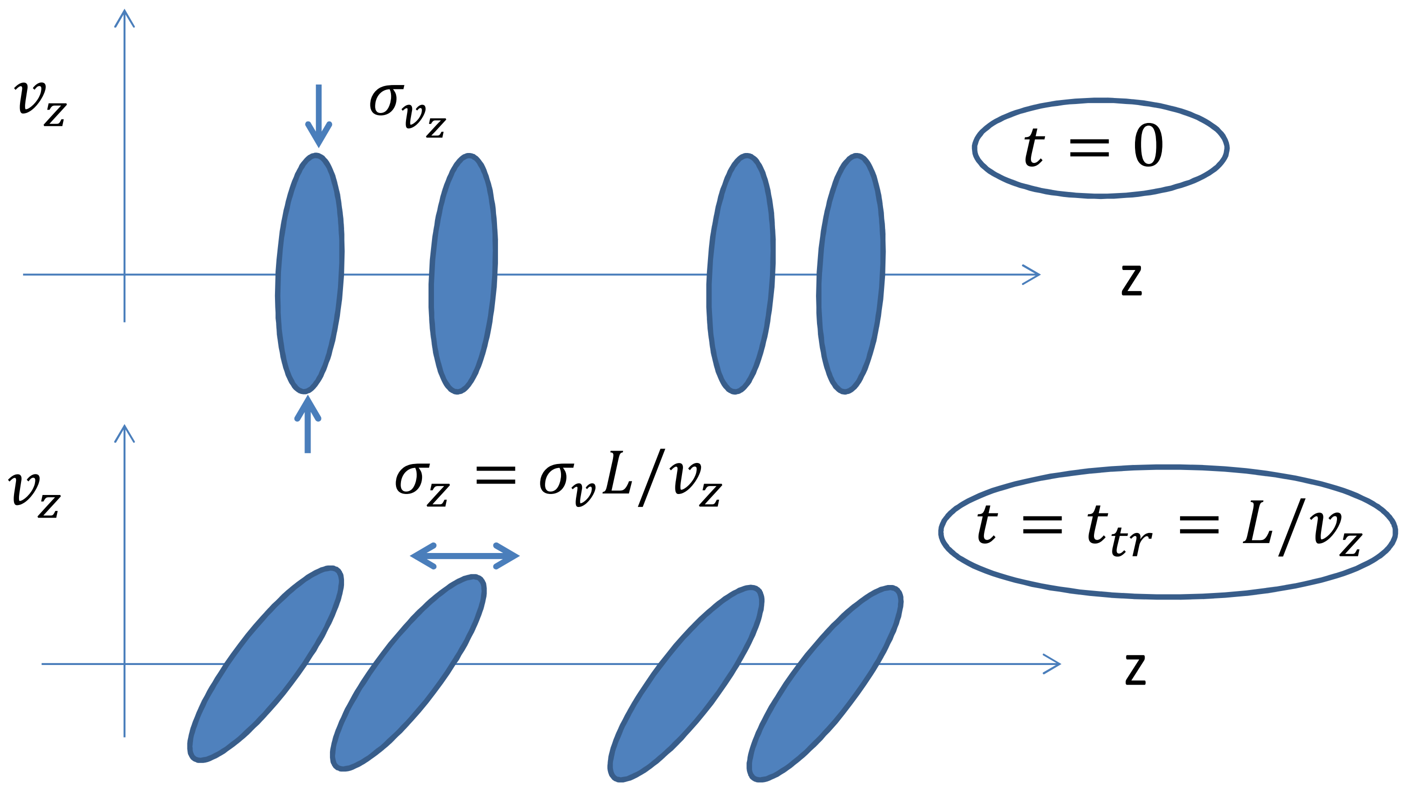
OTR



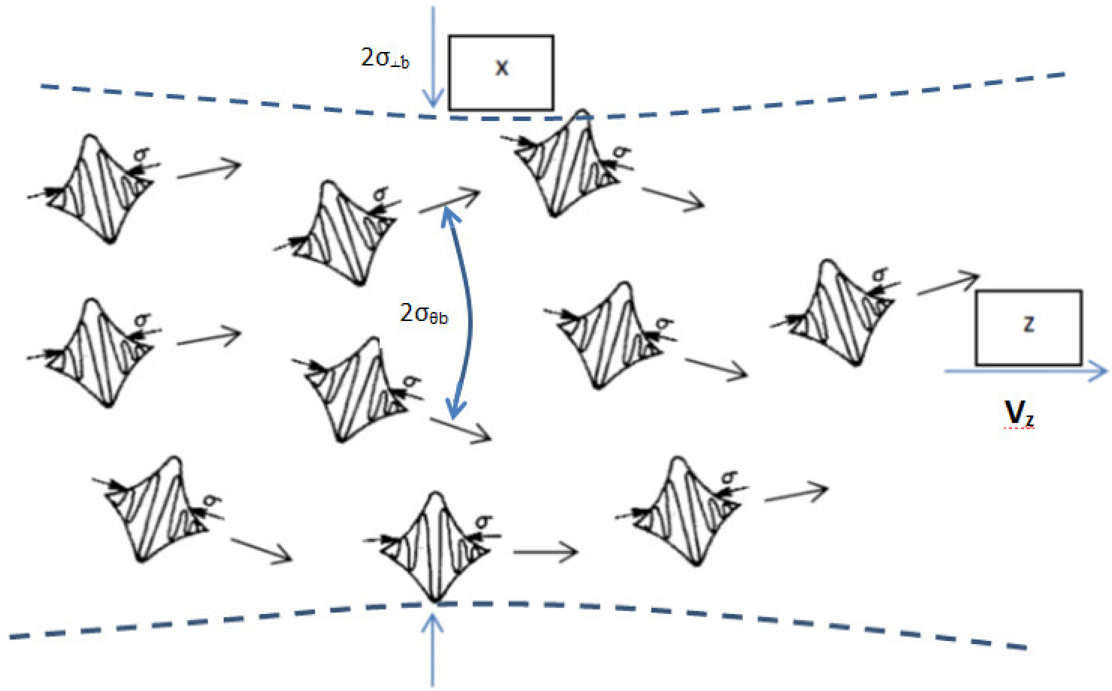
UNDULATOR RADIATION



Particle location uncertainty spread at measurement time t (or drift length L) (Liouville's theorem in phase space)



Fundamental quantum (Heisenberg) particle location uncertainty



$$(\sigma_{||})_{min} = \sqrt{\frac{\lambda_c L}{4\pi\beta\gamma^3}} \qquad (\sigma_{\perp})_{min} = \sqrt{\frac{\lambda_c L}{4\pi\beta\gamma}}$$

A. Friedman, A. Gover, S. Ruschin, G. Kurizki, A. Yariv, Reviews of Modern Physics, 60, 471-535 (April 1988)

Averaging Parseval Theorem over position uncertainty of each particle

$$f(z_j) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(z_j-\bar{z}_j\right)^2/2\sigma^2} \rightarrow N(z) = \sum_{j=1}^{N} f(z - z_j)$$

$$p(k) = \frac{1}{2\pi} e^{-\sigma^2 k^2} \left[N + \sum_{j \neq k}^{N} e^{-ik(\bar{z}_j - \bar{z}_k)} \right]$$

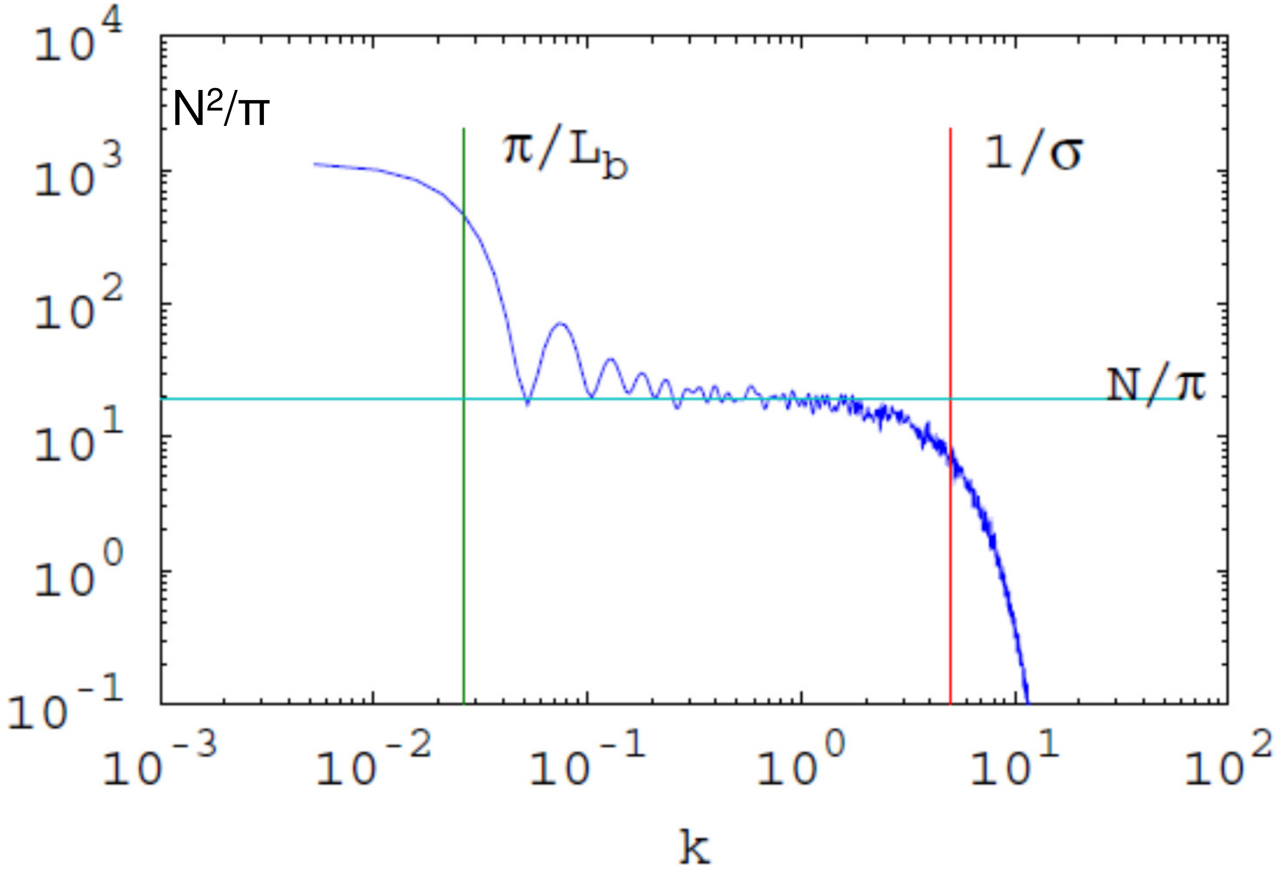
If the particle central locations $\bar{z}_j$ are random (shot-noise):

$$\boxed{[p(k)]_{shot} = \frac{N}{2\pi} e^{-\sigma^2 k^2}}$$

$$E = \int_{-\infty}^{\infty} [p(k)]_{shot} dk = \frac{N}{2\sqrt{\pi}\sigma} \neq \infty$$

The shot-noise cut-off limit is not a property of the beam only: It depends on the measurement (or fundamental) limits.

Spectral Energy of a random electron beam of length L_b and position uncertainty σ



Log-log scale drawing . N=60 random particles in a bunch length L_b=120, uncertainty width σ=0.2

The spectral sum-rule
(Alex Chao)

Back to Parseval:

$$E = \int_{-\infty}^{\infty} \left| \sum_{j}^{N} \delta(z - z_j) \right|^2 dz \quad = \quad \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \sum_{j}^{N} e^{-ikz_j} \right|^2 dk$$

$$\int_{-\infty}^{\infty} \sum_{j=1}^{N} \delta^2(z - z_j) dz + \int_{-\infty}^{\infty} \sum_{i \neq j}^{N} \delta(z - z_j)\delta(z - z_i) dz \quad =$$

$$= \int_{-\infty}^{\infty} p\,(k) dk = \boldsymbol{const.}$$

Validity of the Sum-rule: a Correlated Beam with Position Uncertainty

Averaging Parseval:

$$E = \int_{-\infty}^{\infty} [\sum_{j=1}^{N} f\left(z - z_j\right)]^2 dz = \int_{-\infty}^{\infty} p\,(k) dk = const. \quad (?)$$

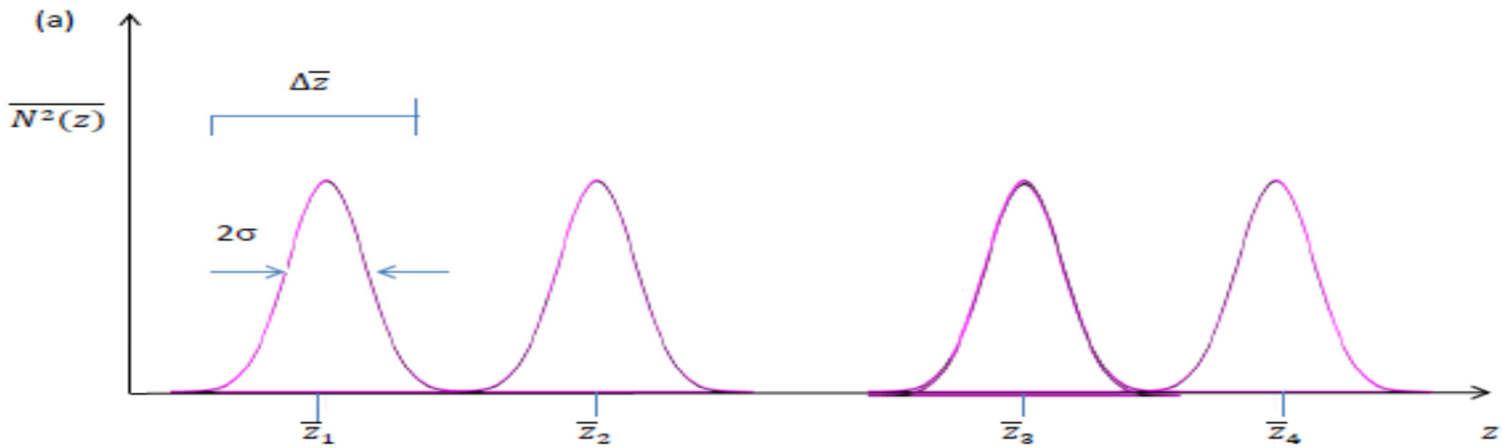
$$\overline{E} = \frac{N}{2\sqrt{\pi}\sigma} + \sum_{i \neq j}^{N} \frac{1}{2\pi^2} \int_{-\infty}^{\infty} e^{-(z-\overline{z}_i)^2/2\sigma^2} e^{-\left(z-\overline{z}_j\right)^2/2\sigma^2}\, dz$$

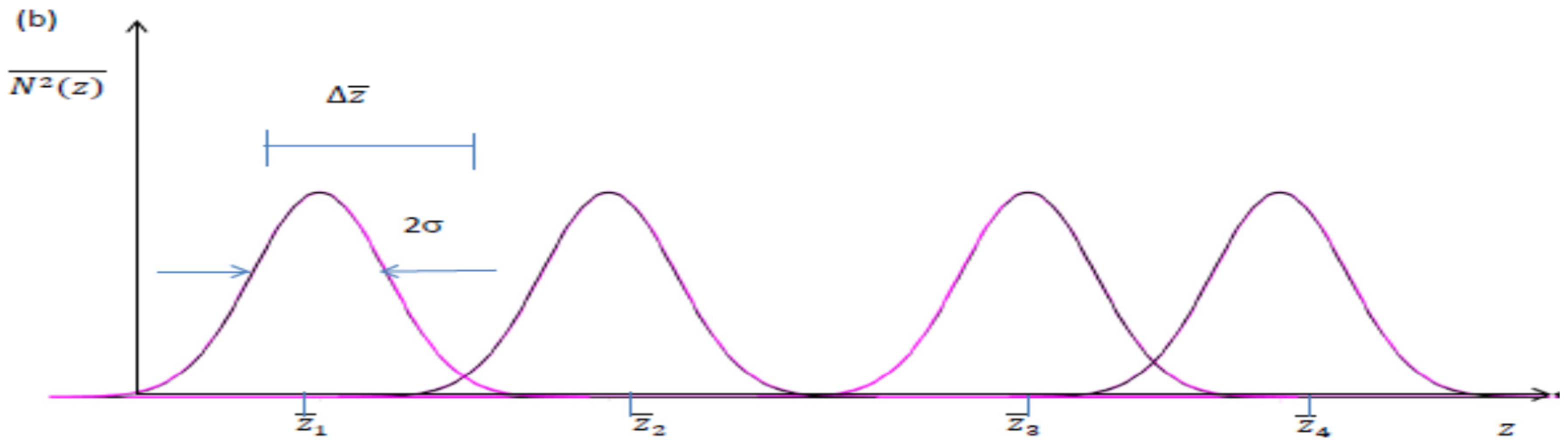
The spectral sum rule is valid if:

Average spacing:

$$\lambda \gg \overline{\Delta z} = \frac{L_b}{N} = \frac{v_z}{I_0/e} \gg \sigma$$

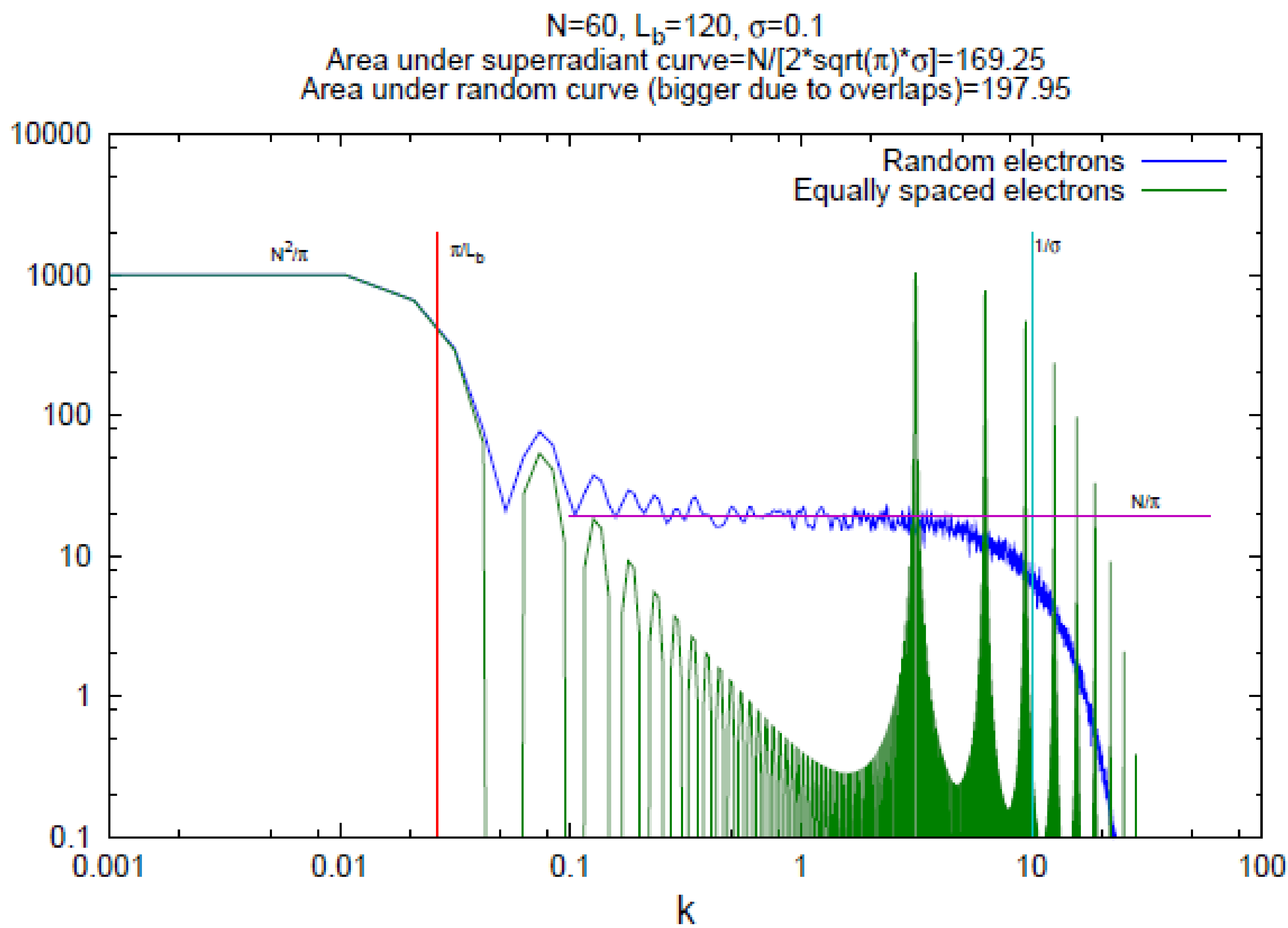
Particles Position-Uncertainty Packets





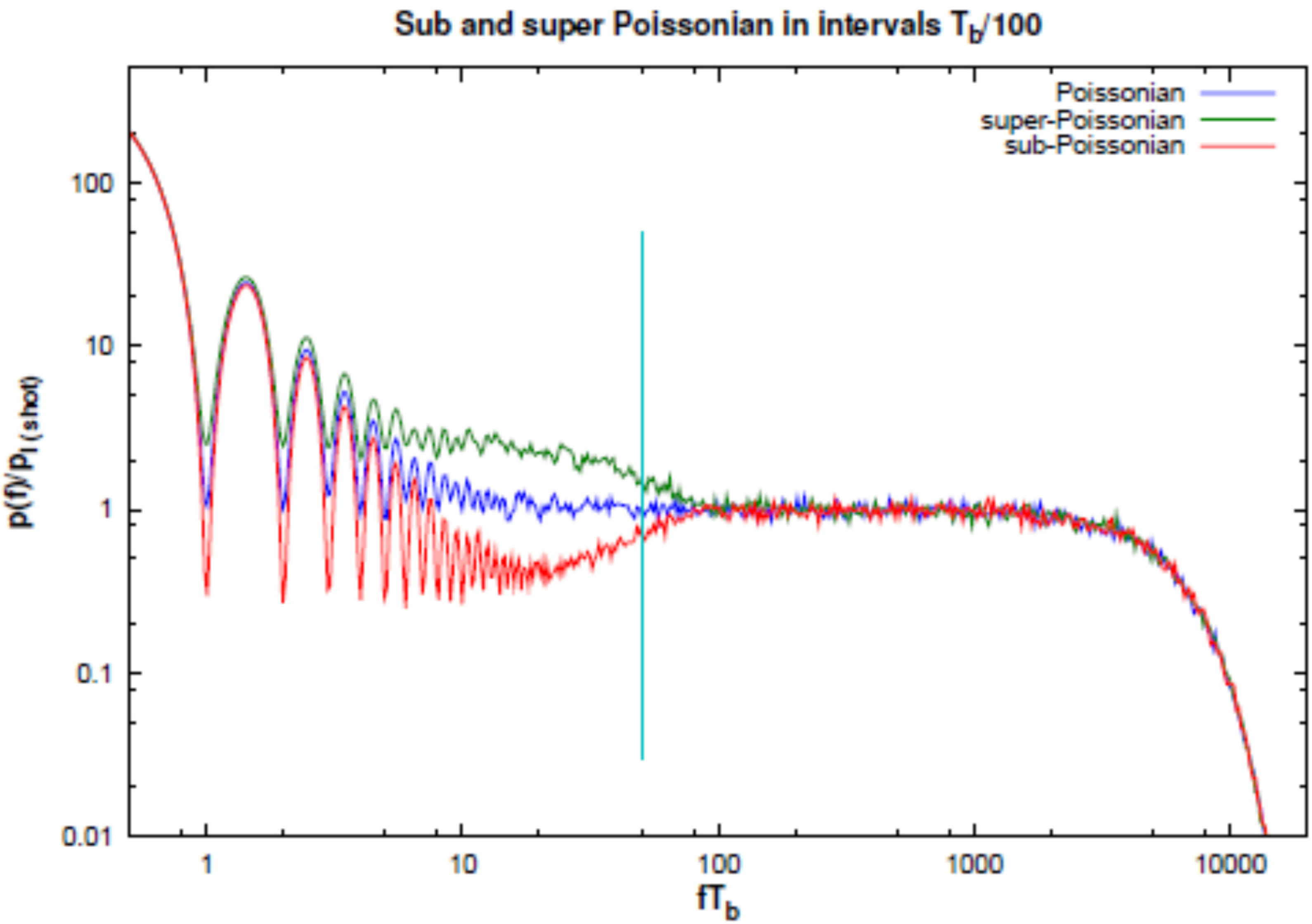
a) Tenuous beam (no overlap): $\Delta\bar{z} > \sigma$

b) Dense beam (overlap): $\Delta\bar{z} < \sigma$.



N=60, L_b=120, σ=0.1
Area under superradiant curve=N/[2*sqrt(π)*σ]=169.25
Area under random curve (bigger due to overlaps)=197.95
Random electrons
Equally spaced electrons
N²/π
π/L_b
1/σ
N/π
10000
1000
100
10
1
0.1
0.001
0.01
0.1
1
10
100
k

The areas under the spectra are 11599, 11392, 11315 for the super-Poissonian, Poissonian, and sub-Poissonian, respectively.



Conclusion (1)

- It is possible to adjust the e-beam current shot- noise level by controlling the longitudinal plasma oscillation dynamics.
- The dispersive transport scheme can be realized with shorter length, but suppression is smaller and the short wavelength limit is tighter.
- Scaling provides advantage to higher beam energies. Suppression at X-UV wavelengths may be feasible. More studies and experiments are needed.
- E-beam noise control can be used to enhance FEL coherence and relax seeding power requirement

Conclusions (2)

- The spectral radiation energy **<u>per radiation mode</u>** is ∝ $p(\omega)$ (the current energy spectral density - ESD). It is:

    -∝ N ($or\ I_0$) (normal spontaneous emission) if the e-beam is random (uncorrelated).

    -Super-radiant if the beam is super-Poissonian.

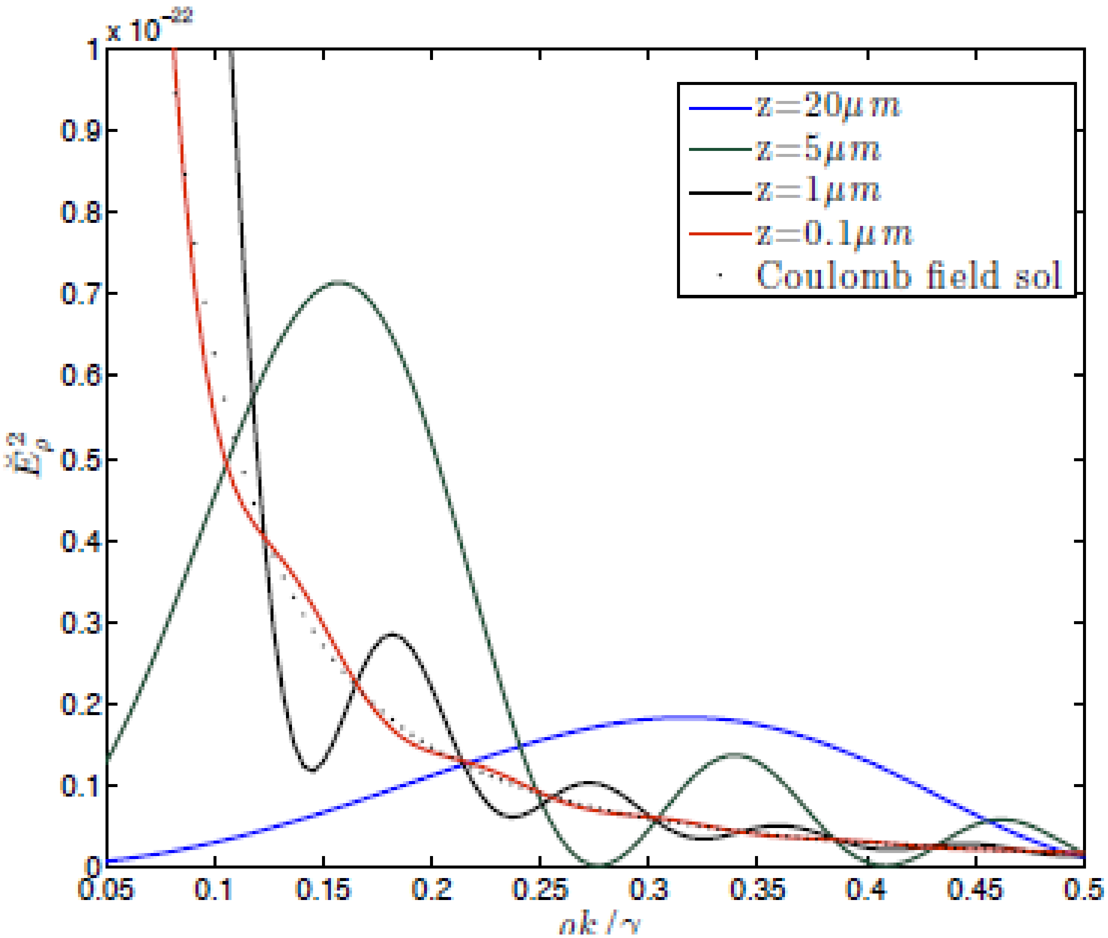
    -Sub-radiant if the beam is sub-Poissonian.

- Undulator radiation measurement is preferable to OTR for measuring beam noise.
- Measured shot-noise spectrum is never "white":

    -It is normally ∝ N ($or\ I_0$) (classical Shot-Noise), it cuts-off for λ<<σ, it is ∝ N^2 for λ>>L_b.

- A spectral sum-rule applies in the range $\overline{\Delta z} \gg \sigma = \lambda_{min}$ (not a practical range for Ampere scale currents)

EXACT OTR SOLUTION IN THE INDUCTIVE NEAR FIELD

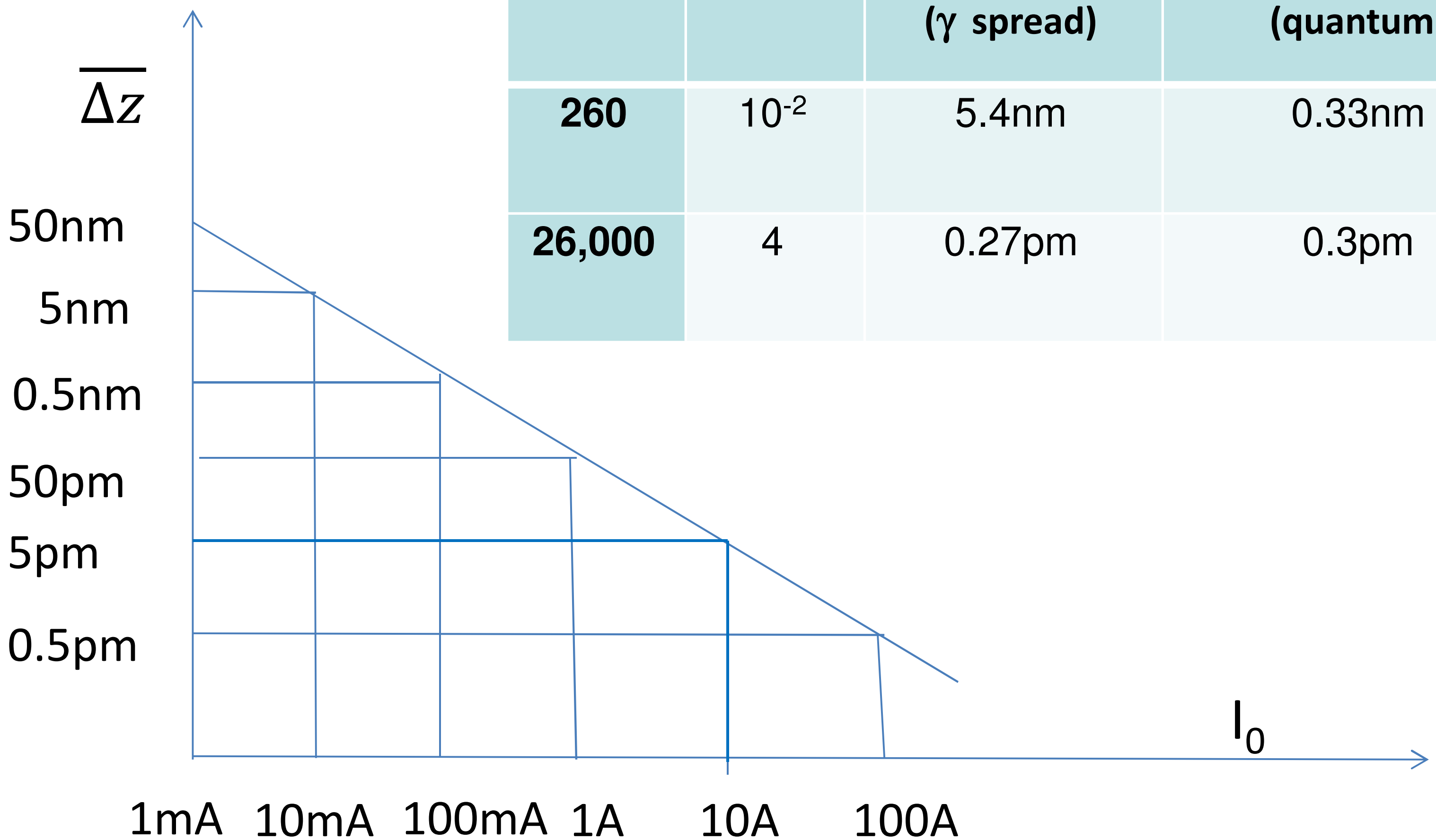


A. Nause, E. Dyunin, R. Ianconescu, A. Gover J. Opt. Soc. Am. B 31, 2438 October 2014

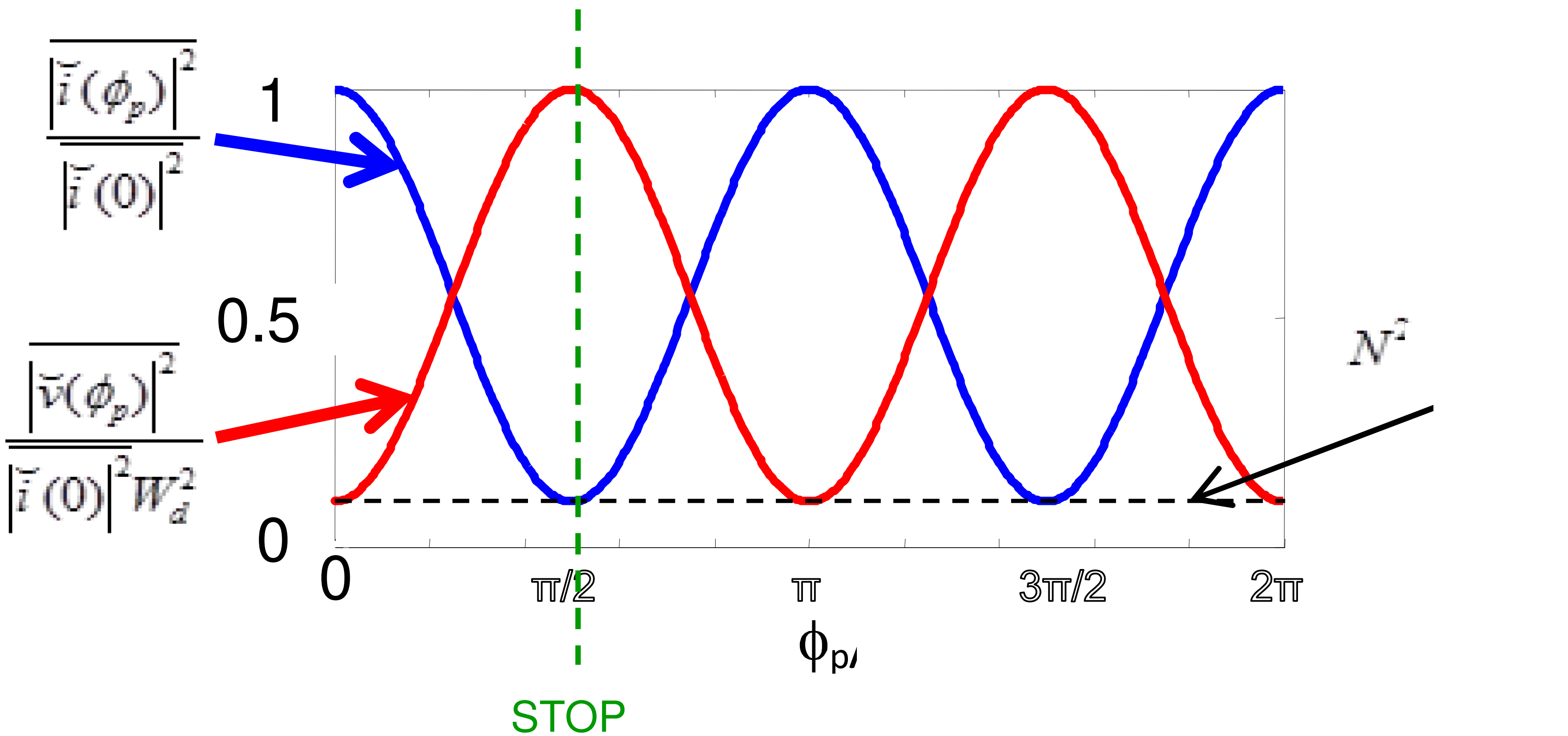
Validity Range of the Spectral Sum-Rule:

$$\overline{\Delta z} = \frac{v_z}{I_0/e} \cong \frac{48pm}{I_0[A]} \gg \sigma = \lambda_{min}$$

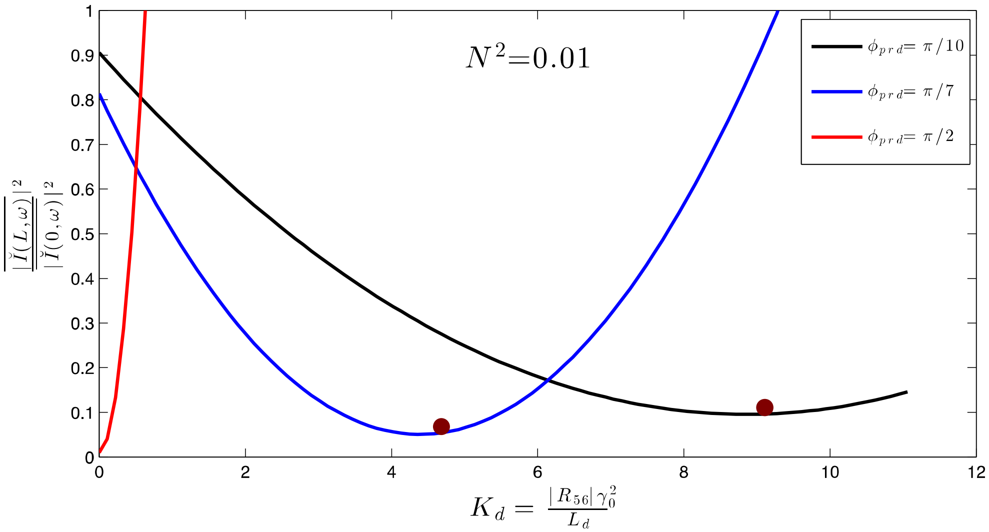
γ	σ_γ	$\sigma(\gamma, L = 1m)$ (γ spread)	$[\sigma(\gamma, L = 1m)]_{min}$ (quantum)
260	10^{-2}	5.4nm	0.33nm
26,000	4	0.27pm	0.3pm

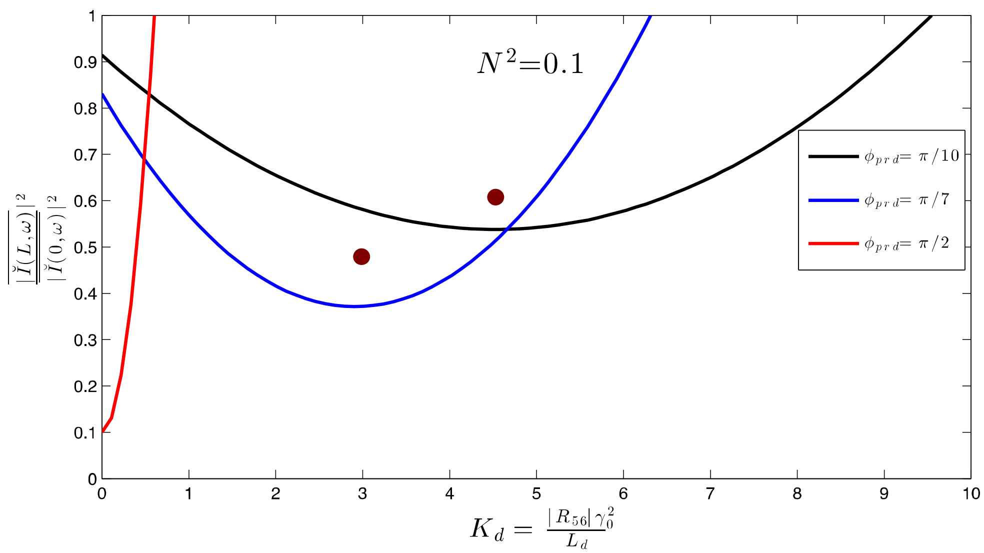


Periodic Power Exchange of Current and Velocity Noise



Dispersive Transport Gain





- Maximal suppression points according to approximation:

$$N^2 << \phi_{pd}^2 << 1$$

Short wavelengths limits

For significant suppression (and negligible Landau damping):

$$N = \frac{\lambda_D}{\lambda} = k\frac{\Delta\beta_z}{\theta_p} << 1$$

Ballistic condition (same as Landau for $L_d=\pi/2\theta_p$):

$$\Delta\phi_p = kL_d\Delta\beta_z << 1$$

SPARC:
Current 50 A
Beam Energy 176 MeV
Beam Radius 150 um
Sliced Energy Spread 10^{-4}
Emittance 1 mm mrad

$L\pi/2 = 14m$

$$\begin{cases} \dfrac{k}{\theta_p}\dfrac{\Delta\gamma}{\gamma^3} << 1 \\ \dfrac{k}{\theta_p}\left(\dfrac{\varepsilon_n}{\gamma\sigma_x}\right)^2 << 1 \end{cases}$$

λ >> 46 nm

λ >> 21 nm

**TUPD17, Proceedings of FEL2012, Nara, Japan*

Granularity condition:

$$n_0 A_e \lambda = \frac{I_0}{ec}\lambda >> 1$$

10,000

(for λ = 10 nm)

SHORT WAVELENGTH LIMIT $k_{max}=2\pi/\lambda_{min}$ FOR DESIRABLE SUPPRESSION G_{min}

	DRIFT	DRIFT + DISPERSION
• Drift phase:	$\varphi_p = \pi/2$	$\varphi_p \ll \pi/2$
• Optimal dispersion	0	$K_d = \frac{\gamma_0^2 \lvert R_{56} \rvert}{L_d} = \frac{1}{\varphi_p^2}$
• G_{min}	N^2	N^2/φ_p^2

$k_{max}=2\pi/\lambda_{min}$ for given G_{min}

DRIFT:

$$\begin{cases} \dfrac{k_m \Delta\gamma}{\theta_p \gamma^3} = G_{min}^{1/2} \\ \dfrac{k_m}{\theta_p}\left(\dfrac{\varepsilon_n}{\gamma\sigma_x}\right)^2 = G_{min}^{1/2} \end{cases}$$

DRIFT + DISPERSION:

$$\begin{cases} \dfrac{k_m \Delta\gamma}{\theta_p \gamma^3} = G_{min}^{1/2}/\varphi_p \\ \dfrac{k_m}{\theta_p}\left(\dfrac{\varepsilon_n}{\gamma\sigma_x}\right)^2 = G_{min}^{1/2}/\varphi_p \end{cases}$$

- Scaling: $\theta_p \propto \gamma^{-3/2} \propto 1/\sigma_x$

(SCALING ADVANTAGE AT HIGH ENERGIES)