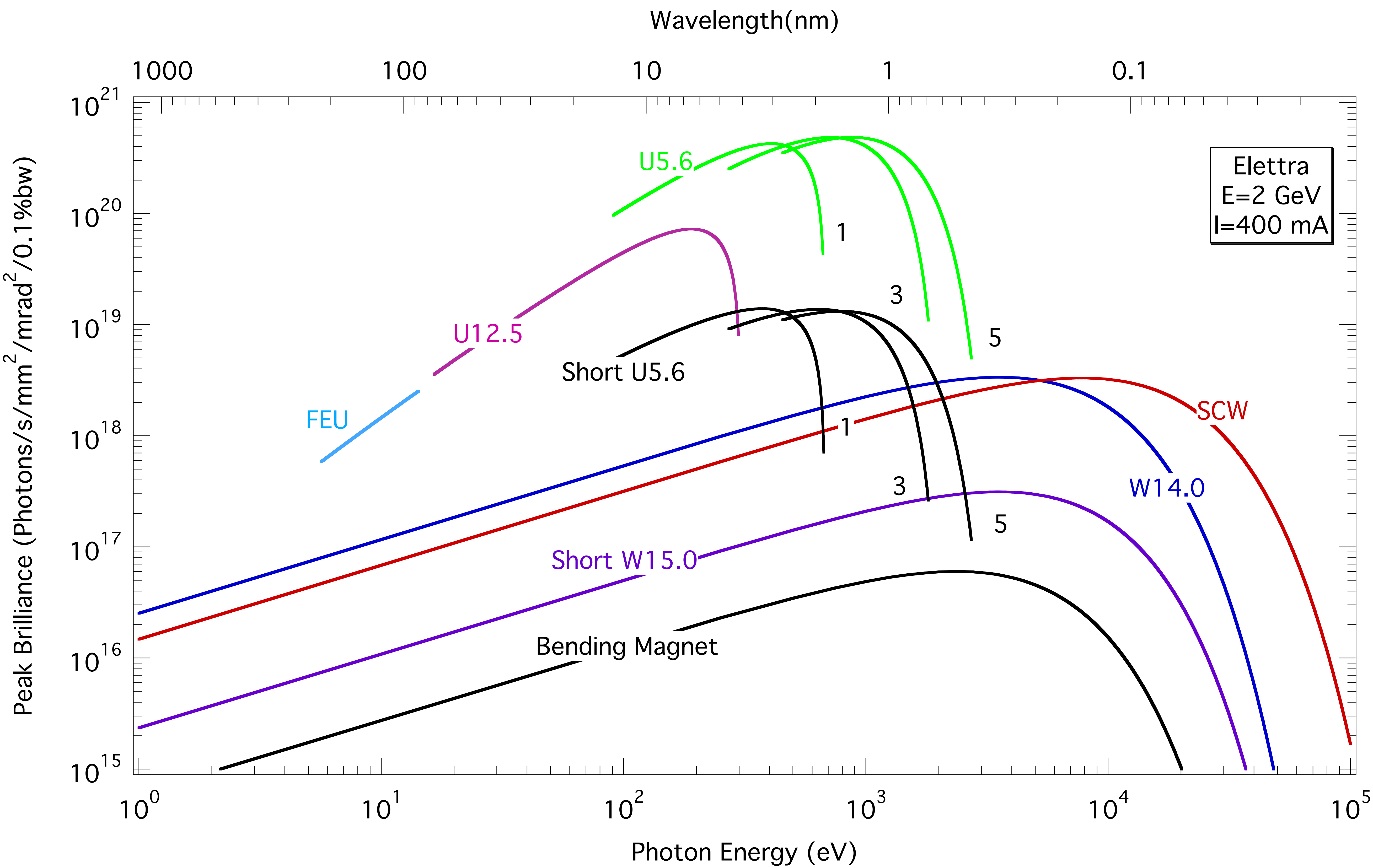
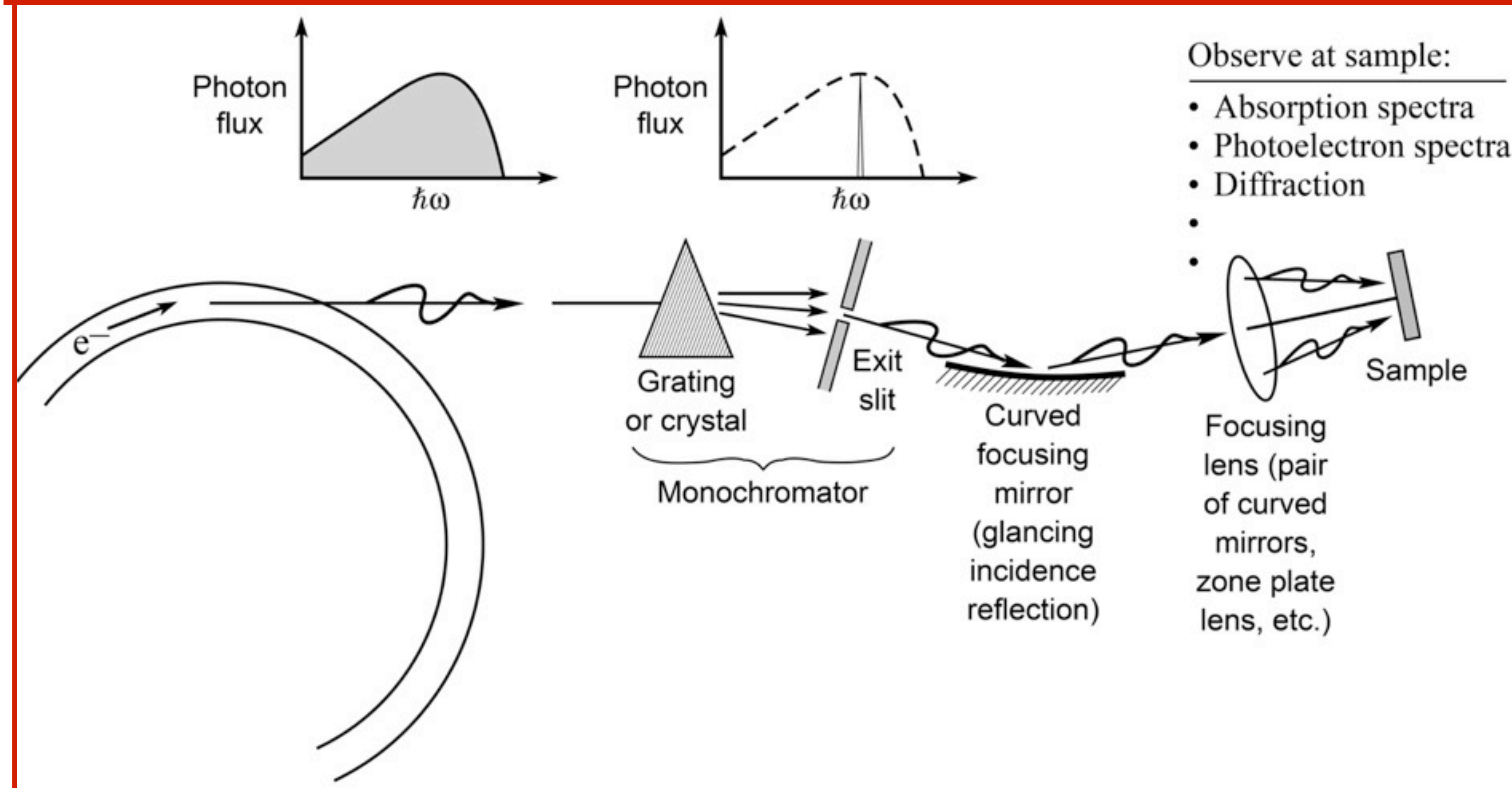
Beamlines

Some of the Photon sources at Elettra



The beamline



- is the mean of bringing radiation from the source to the experiment transforming the phase volume in a controlled way: it demagnifies, monochromatizes and refocuses the source onto a sample

- must preserve the excellent qualities of the radiation source

Brilliance

$$\text{Brilliance} = \frac{\Phi}{\sigma_x \sigma_y \sigma'_x \sigma'_y \text{BW}}$$

where:
Φ is the photon flux
$\sigma_{x,y}$ are the source sizes
$\sigma'_{x,y}$ are the source divergences
BW is the bandwidth

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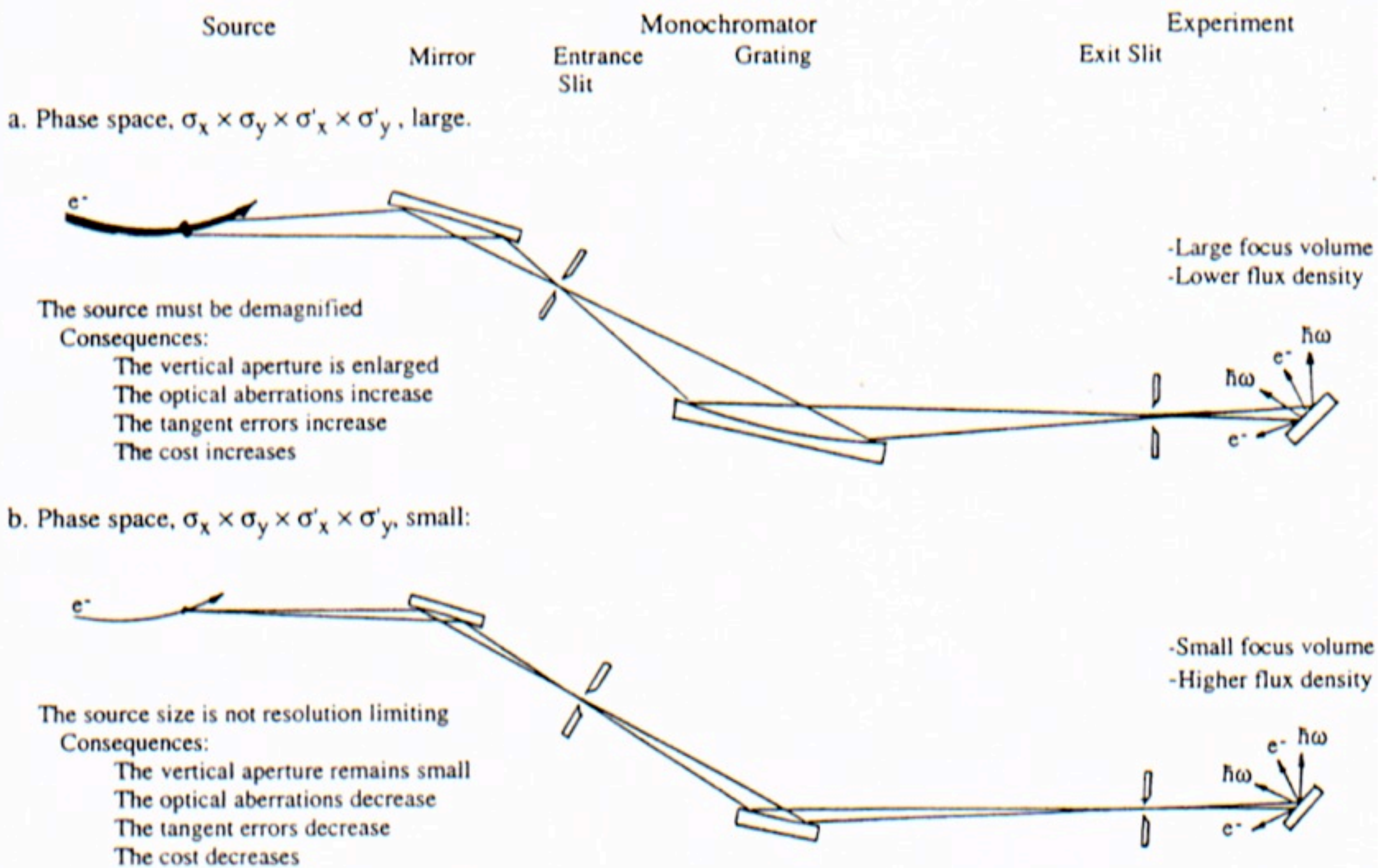
Figure 1.2.1: The Practical Meaning of Brilliance



Brilliance

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where:
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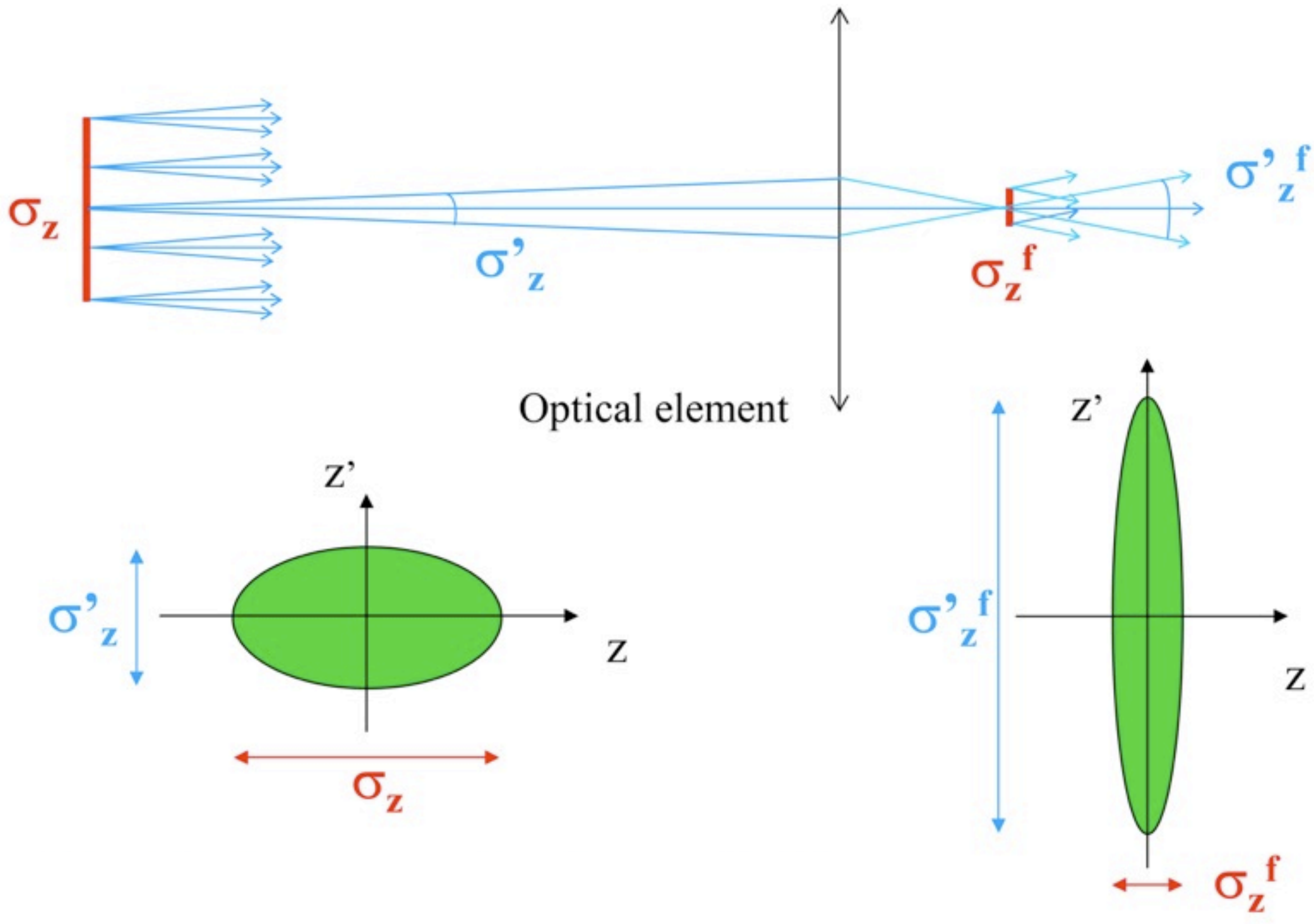
BW is the bandwidth

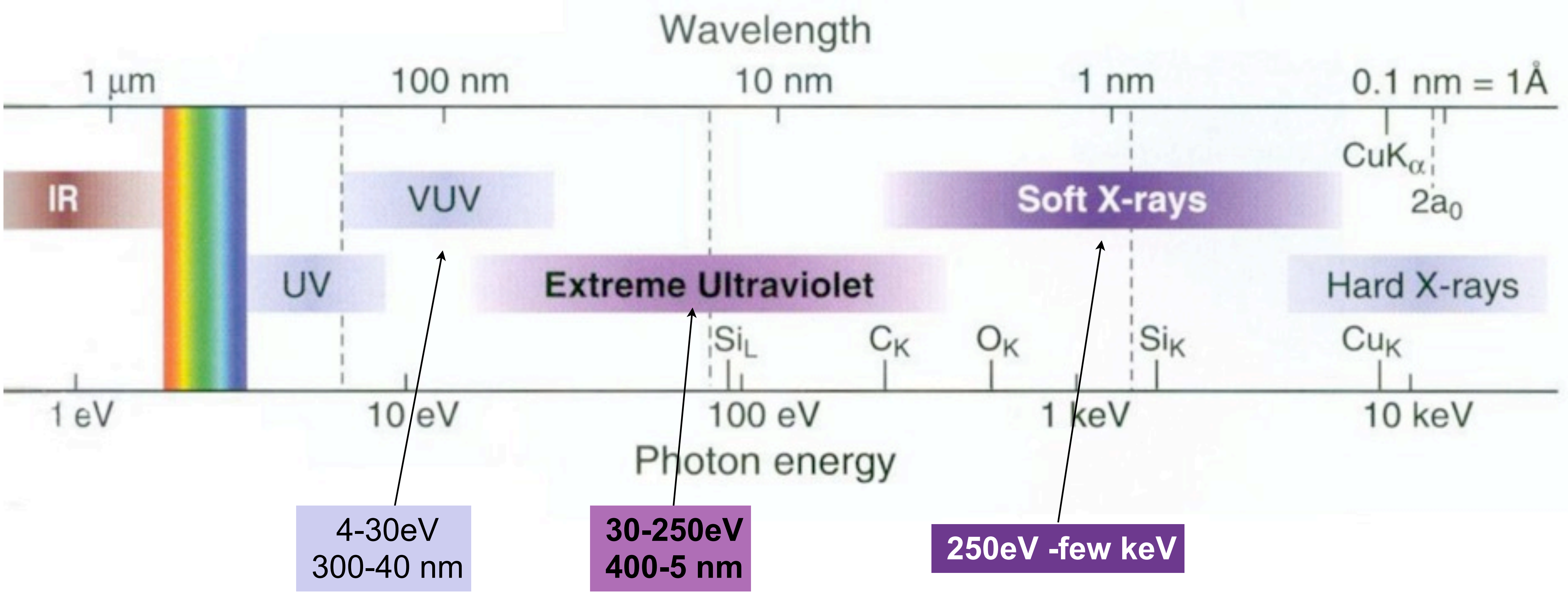
Liouville's theorem
For an optical system the occupied phase space volume can only increase along the optical path (without loosing photons) $(\sigma\sigma')_{final} \geq (\sigma\sigma')_{initial}$

Brilliance

Example a focussing element:
by reducing the size we increase the divergence



VUV, EUV and soft x-rays



These regions are interesting because they are characterized by the presence of the absorption edges of most low and intermediate Z elements ➔ photons with these energies are a very sensitive tool for elemental and chemical identification

But… these regions are difficult to access.

VUV, EUV and soft x-rays

Ultra-high vacuum

VUV, EUV and soft x-rays have a high degree of absorption in all materials

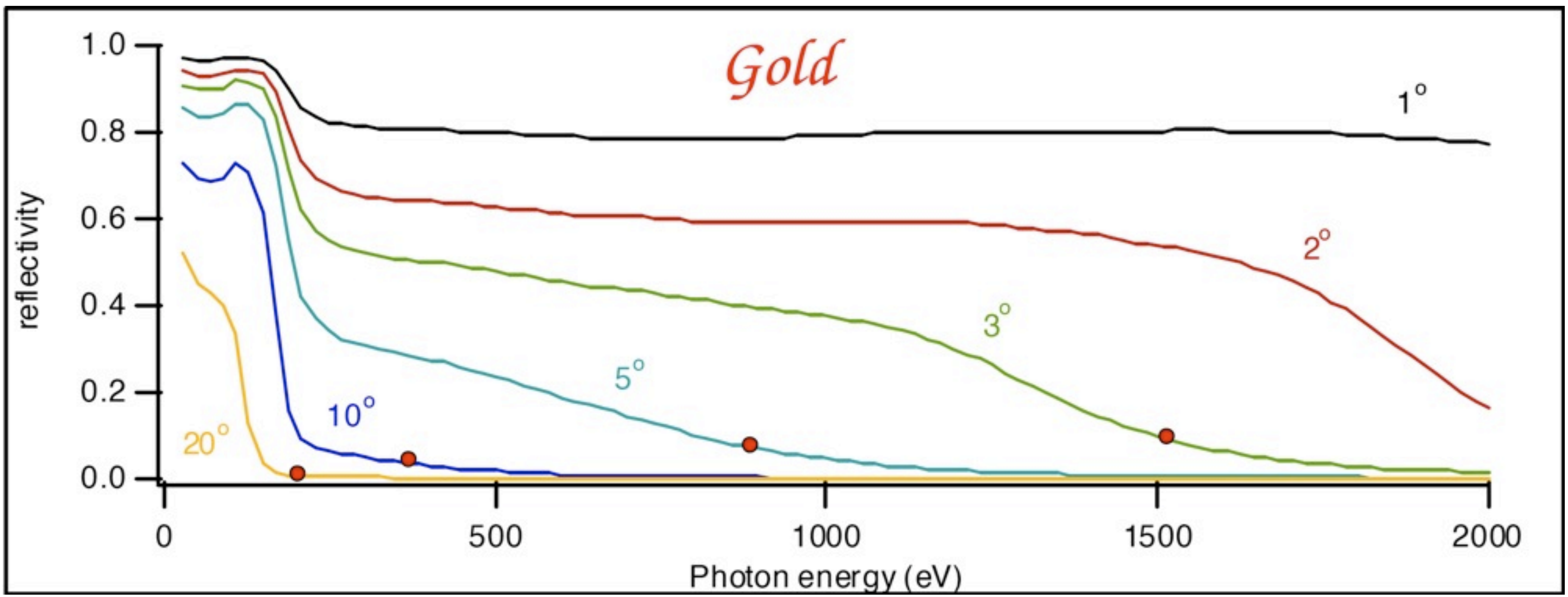
➔ No windows
➔ The entire optical system must be kept under vacuum

Ultrahigh vacuum conditions ($P \lesssim 1\text{-}2 \times 10^{-9}$ mbar) are required:
• Not to disturb the storage ring and the experiment
• To avoid photon absorption in air
• To protect the optical surfaces from contamination (especially from carbon)

VUV, EUV and soft x-rays

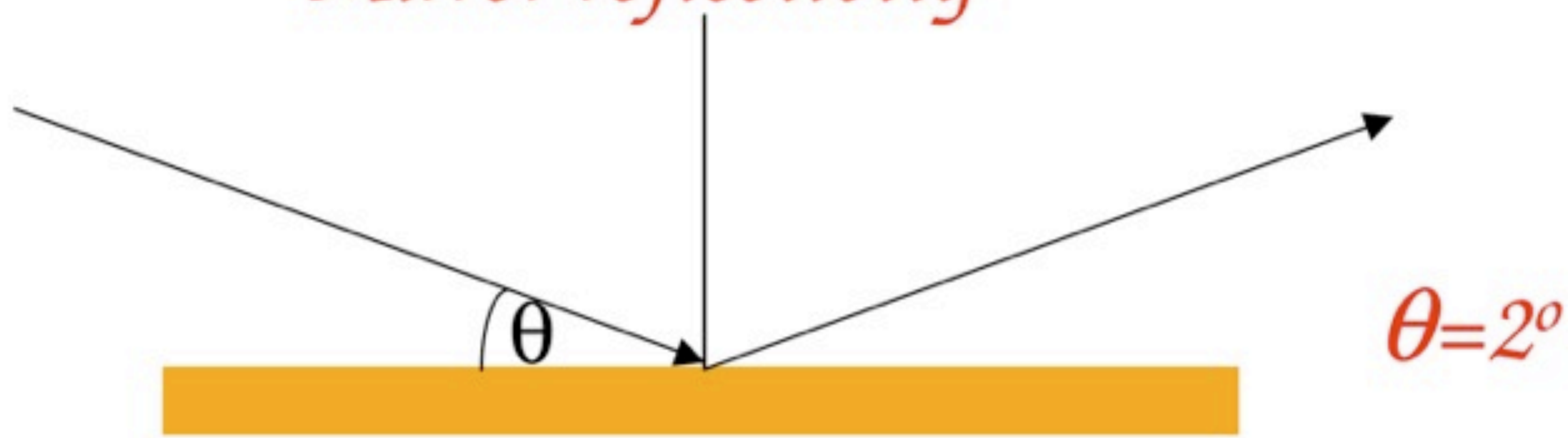
VUV, EUV and soft x-rays have a high degree of absorption in all materials
➔ No lenses: only mirrors!

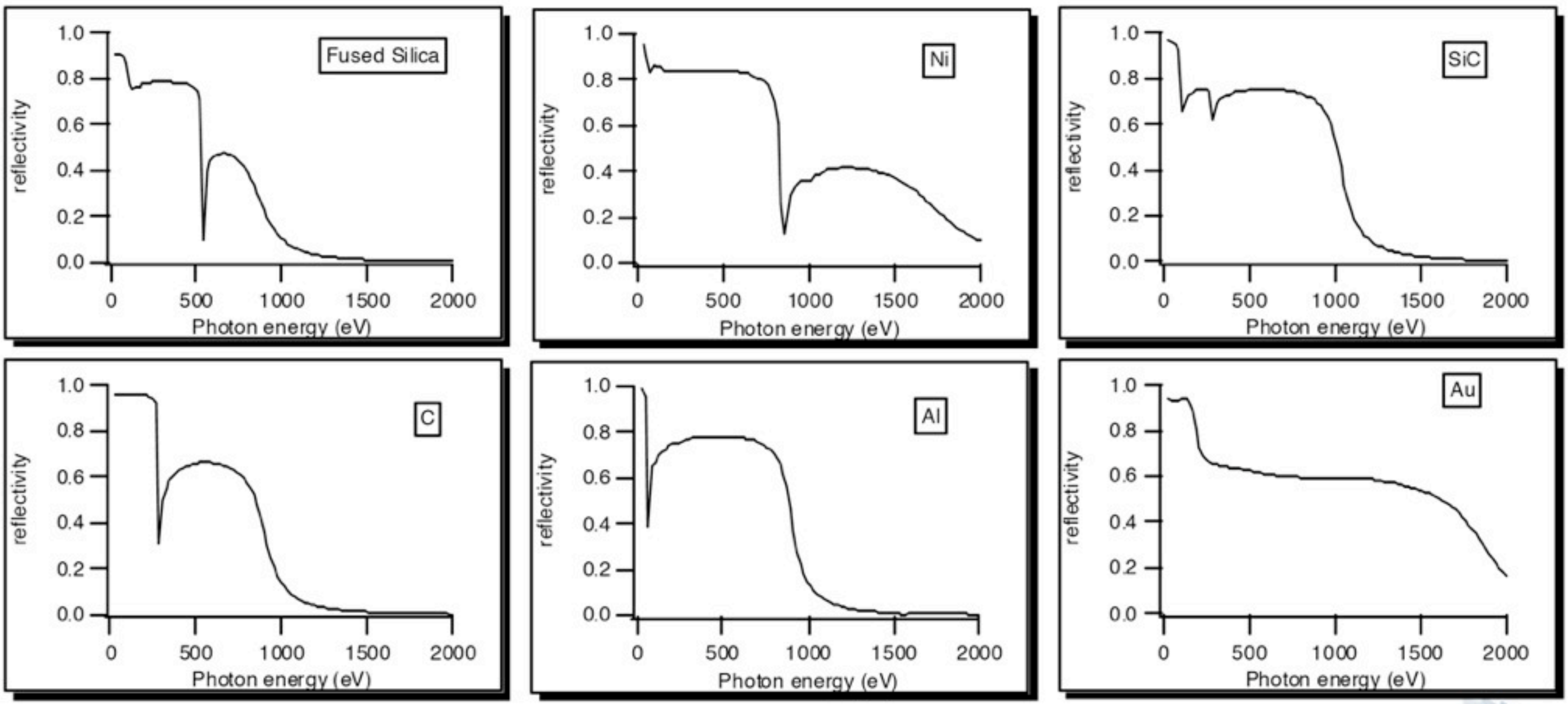
Reflectivities drop down by increasing the grazing incidence angle
➔ only reflective optics at grazing incidence angles (1-2 degrees)



VUV, EUV and soft x-rays

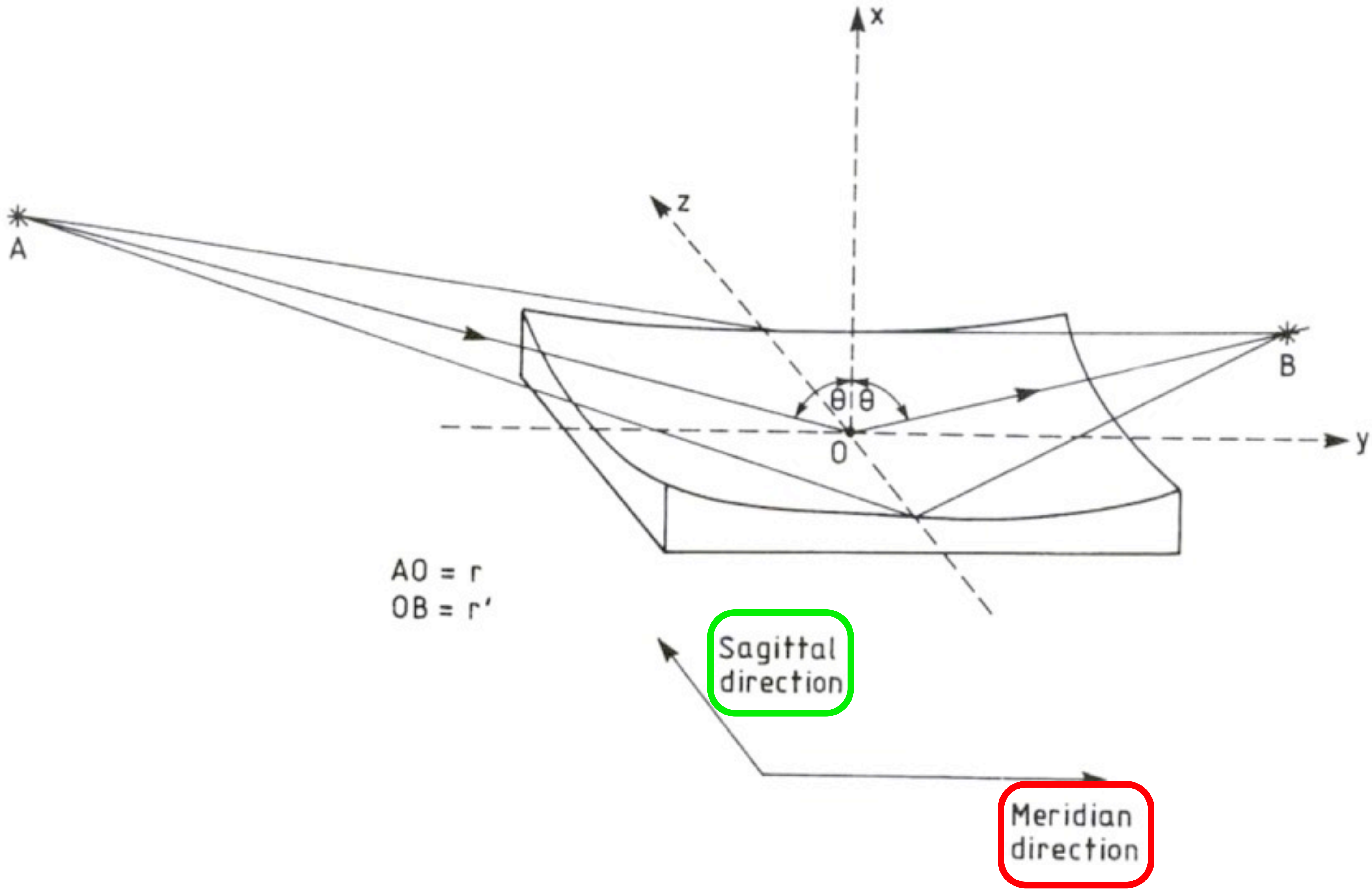
Mirror reflectivity





Focusing properties

The meridian or tangential plane contains the central incidence ray and the normal to the surface. The sagittal plane is the plane perpendicular to the tangential plane and containing the normal to the surface.



Paraboloid

Rays traveling parallel to the symmetry axis OX are all focused to a point A.

Conversely, the parabola collimates rays emanating from the focus A.

Line equation: $Y^2 = 4aX$

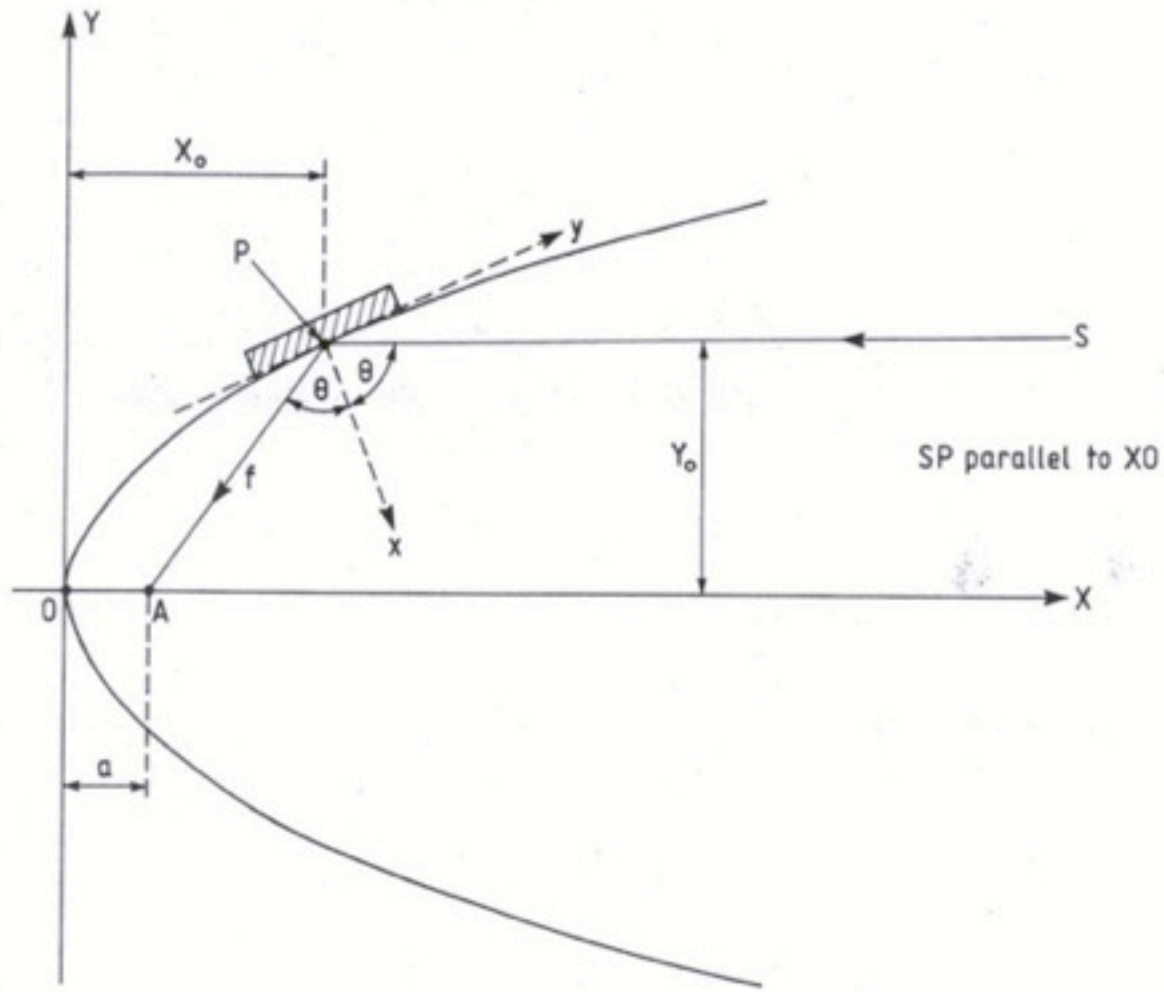
Paraboloid equation: $Y^2 + Z^2 = 4aX$

where: $a = f\cos^2\vartheta$

Position of the pole P:

$$X_o = a\tan^2\vartheta$$

$$Y_o = 2a\tan\vartheta$$



Paraboloid equation:

$$x^2\sin^2\vartheta + y^2\cos^2\vartheta + z^2 - 2xy\sin\vartheta\cos\vartheta - 4ax\sec\vartheta = 0$$

J.B. West and H.A. Padmore, Optical Engineering, 1987

Ellipsoid

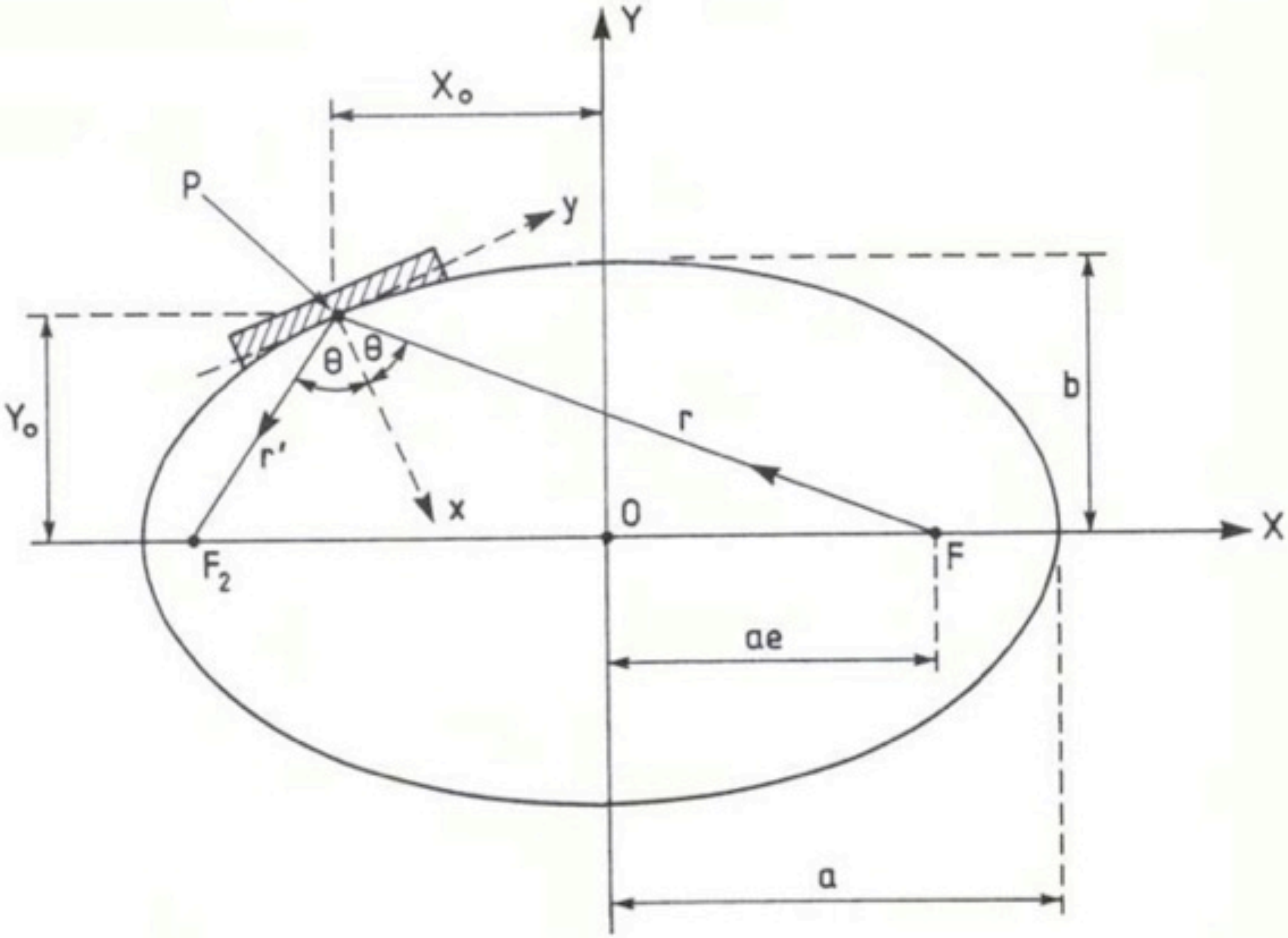
Line equation: $\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1$

Ellipsoid equation:

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}+\frac{Z^2}{b^2}=1$$

where: $a=\frac{r+r'}{2}$; $b=a\sqrt{1-e^2}$

$$e=\frac{1}{2a}\sqrt{r^2+r'^2-2rr'\cos(2\vartheta)}$$



Rays from one focus F_1 will always be perfectly focused to the second focus F_2.

$$x^2\left(\frac{\sin^2\vartheta}{b^2}+\frac{1}{a^2}\right)+y^2\left(\frac{\cos^2\vartheta}{b^2}\right)+\frac{z^2}{b^2}-x\left(\frac{4f\cos\vartheta}{b^2}\right)-xy\left[\frac{2\sin\vartheta\sqrt{e^2-\sin^2\vartheta}}{b^2}\right]=0$$

where: $f=\left(\frac{1}{r}+\frac{1}{r'}\right)^{-1}$

J.B. West and H.A. Padmore, Optical Engineering, 1987

Toroid (1)

The bicycle tyre toroid is generated by rotating a circle of radius ρ in an arc of radius R. In general, two non-coincident focii are produced: one in the meridional plane and one in the sagittal plane

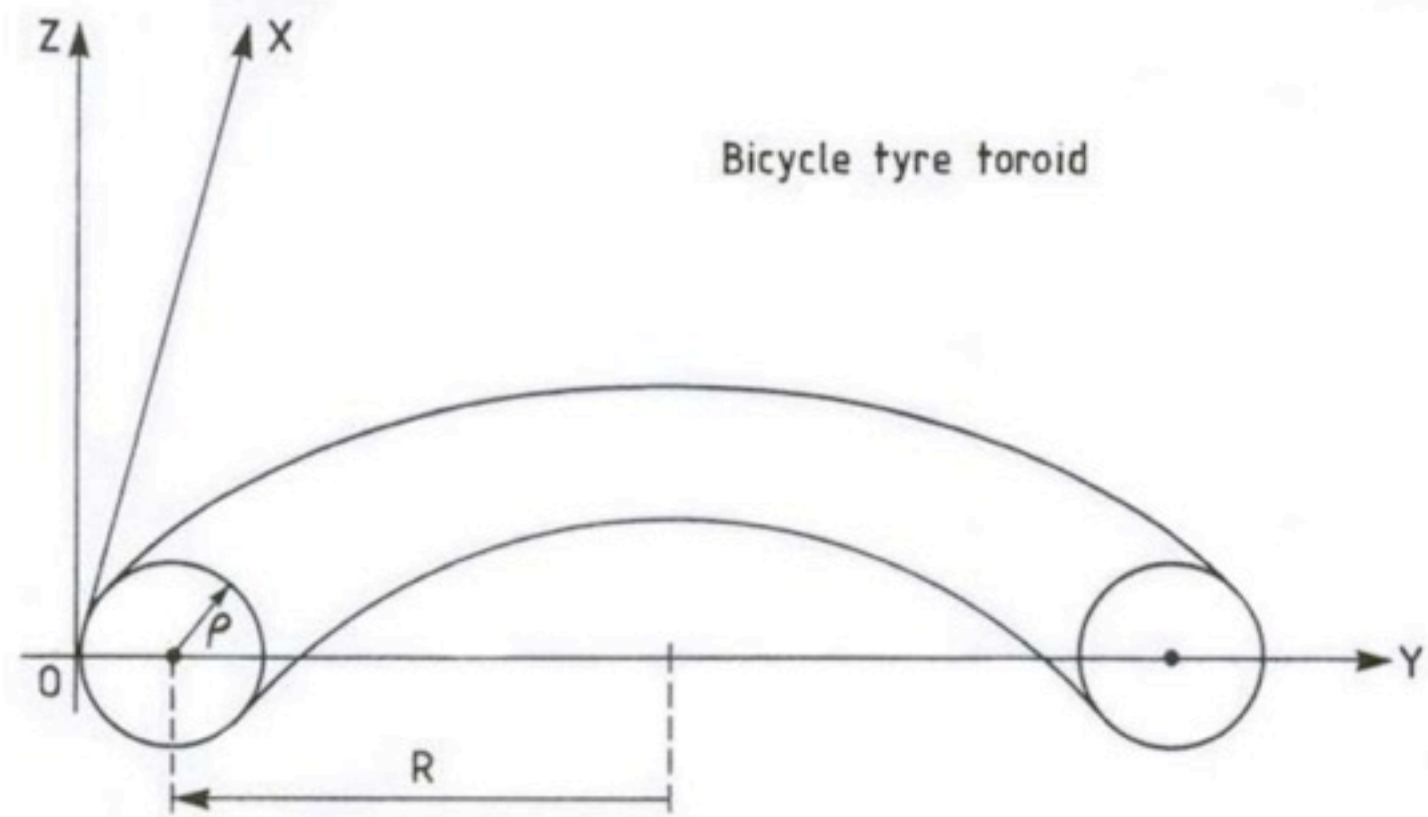
Tangential focus:

$$\left(\frac{1}{r}+\frac{1}{r'_t}\right)\frac{\cos\vartheta}{2}=\frac{1}{R}$$

Sagittal focus:

$$\left(\frac{1}{r}+\frac{1}{r'_s}\right)\frac{1}{2\cos\vartheta}=\frac{1}{\rho}$$

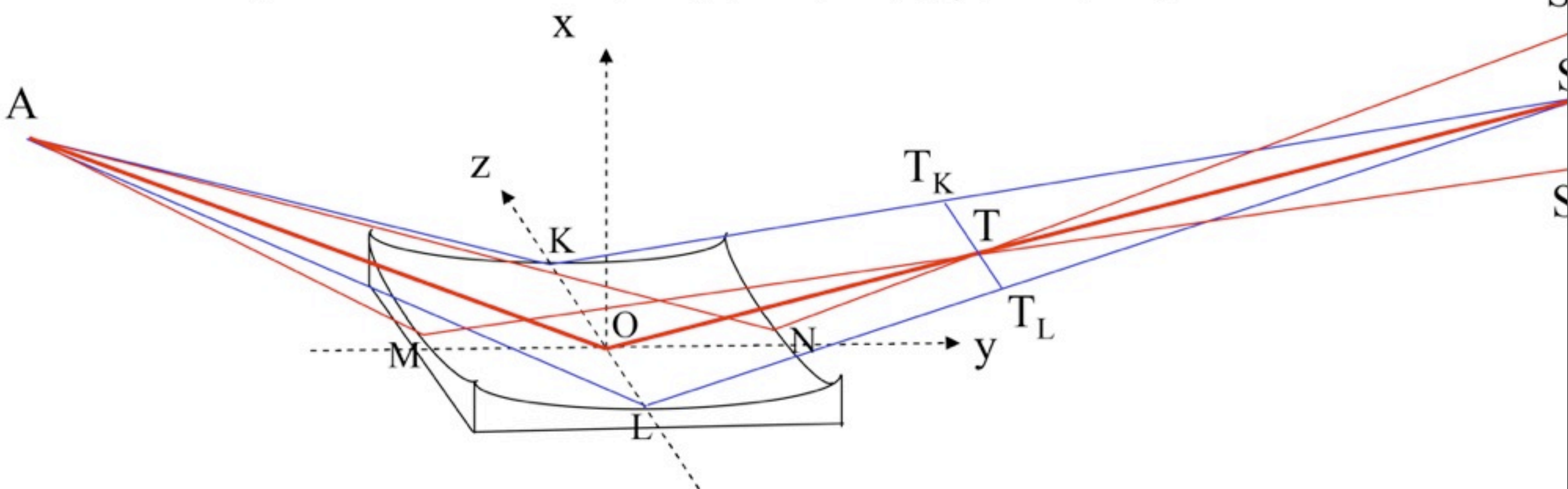
Stigmatic image: $\dfrac{\rho}{R}=\cos^2\vartheta$



J.B. West and H.A. Padmore, Optical Engineering, 1987

Toroid (2)

$$x^2 + y^2 + z^2 = 2Rx - 2R(R-\rho) + 2(R-\rho)\sqrt{(R-x)^2 + y^2}$$

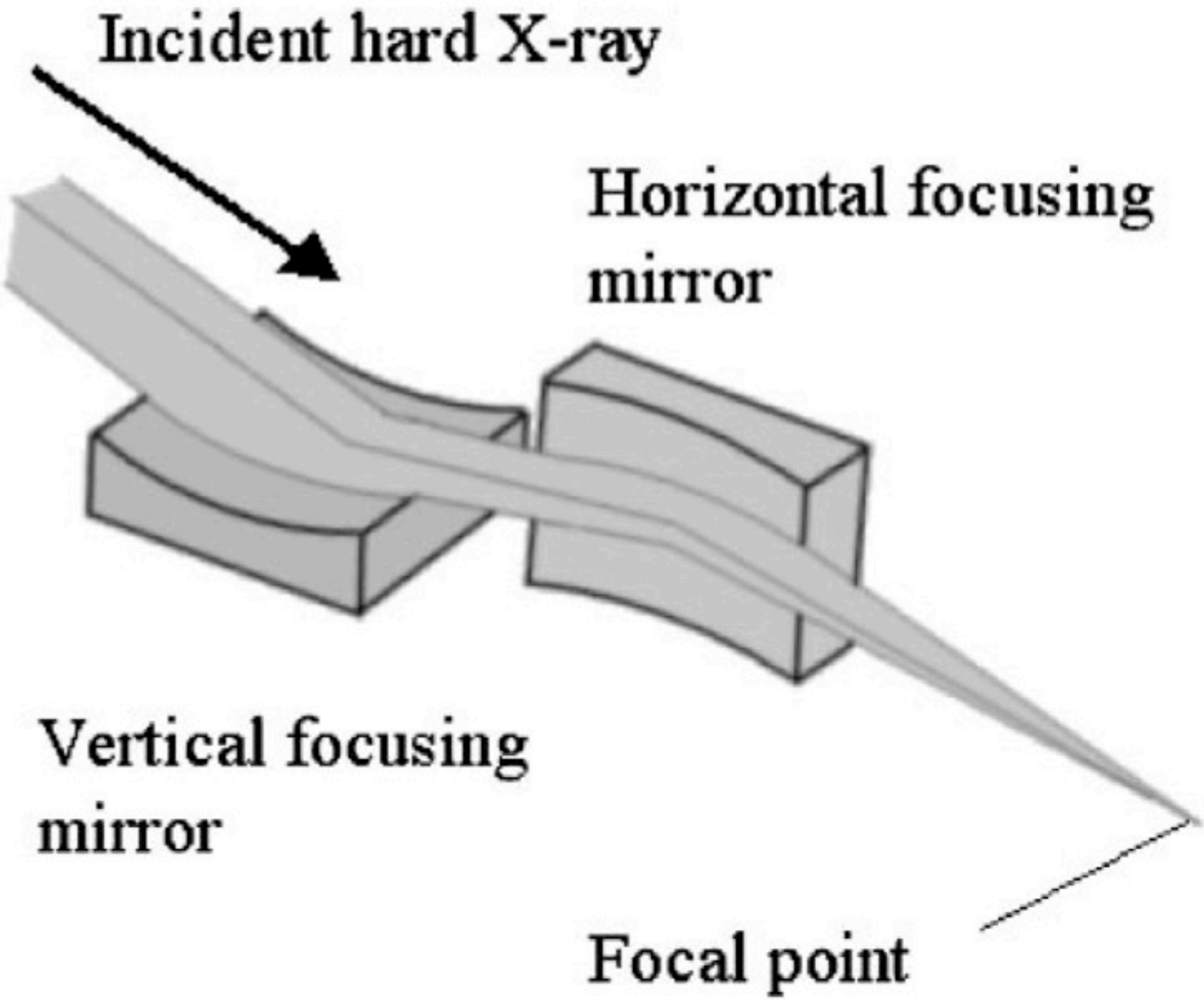


For ρ=R → spherical mirror

A stigmatic image can only be obtained at normal incidence.

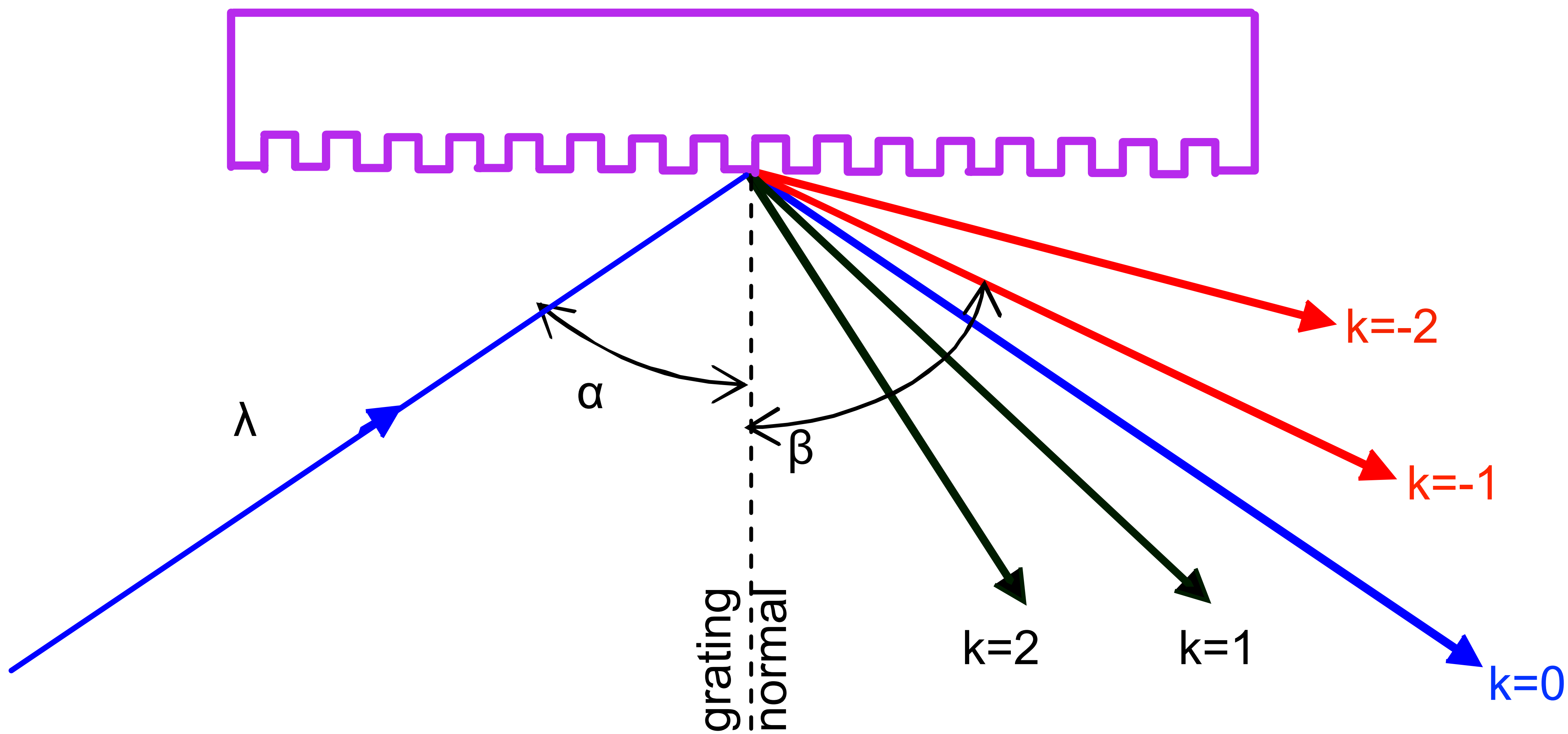
For a vertical deflecting spherical mirror at grazing incidence the horizontal sagittal focus is always further away from the mirror than the vertical tangential focus. The mirror only weakly focusses in the sagittal direction.

The Kirkpatrick-Baez spherical mirrors configuration



Gratings

The diffraction grating separates the different components of the spectrum by redirecting the radiation by an amount which depends upon the wavelength.



$$\sin\alpha + \sin\beta = Nk\lambda$$

N=1/d is the groove density, k is the order of diffraction (±1,±2,...)

VUV, EUV and soft x-rays beamline

Basic elements:

- mirrors to focus, deflect and filter
- gratings to diffract
- slits to spatially select the radiation

Optical elements have to preserve (as much as possible!) **the quality (brilliance) of the radiation**

Conserving brilliance

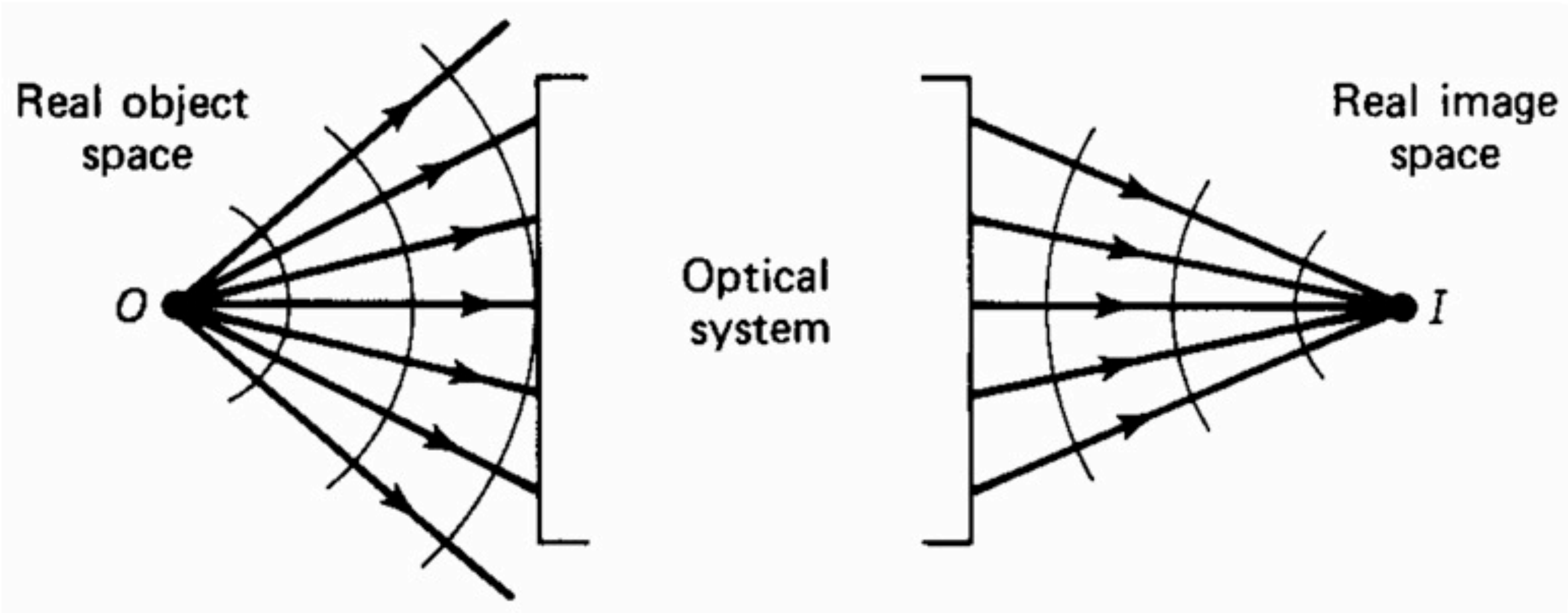
Brilliance decreases because of:

- roughness and slope errors on optical surfaces
- thermal deformations of optical elements due to heat load produced by the high power radiation
- aberrations of optical elements

In the following we will consider OEs with theoretical surface shapes

Perfect imaging and aberrations

An ideal optical element is able to perform perfect imaging if all the rays originating from a single object point cross at a single image point.



Deviations from perfect imaging are called aberrations.

Aberration theory

Image quality is essential for achieving high energy and spatial resolution ➔ knowledge of aberration theory is necessary

It shows what the different aberration terms are and how they play a role in the image formation ➔ it teaches how aberrations can be reduced

Goal: understand in general terms how to treat mathematically the focusing properties of a concave optical element.

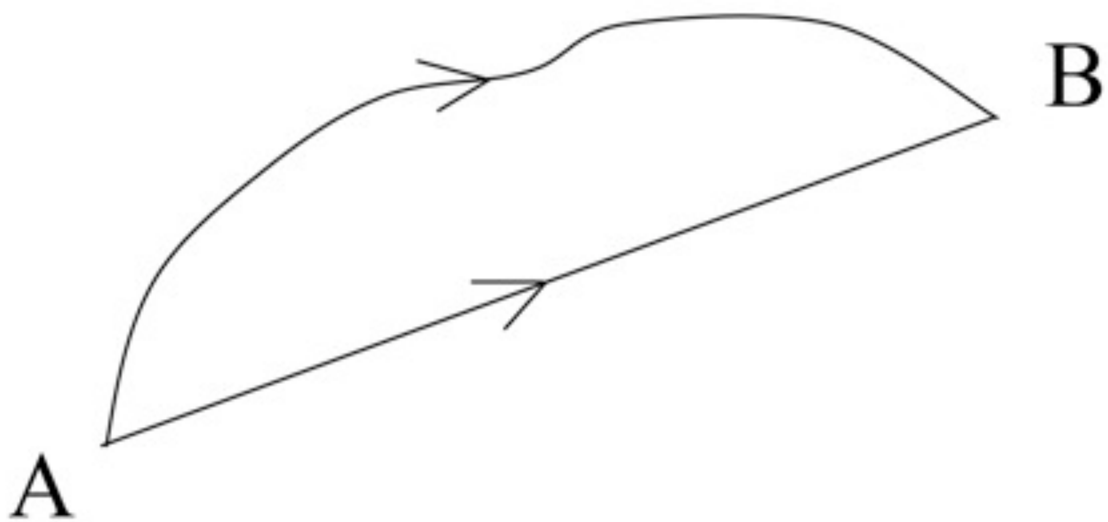
We will study the case of a grating.

The general theory of aberrations of diffraction gratings applies Fermat's principle to derive expressions for the aberration coefficients.

Fermat's principle

Light-rays choose their paths to minimize the optical length

$$F = \int_{A}^{B} n\left(\vec{r}\right) dl$$



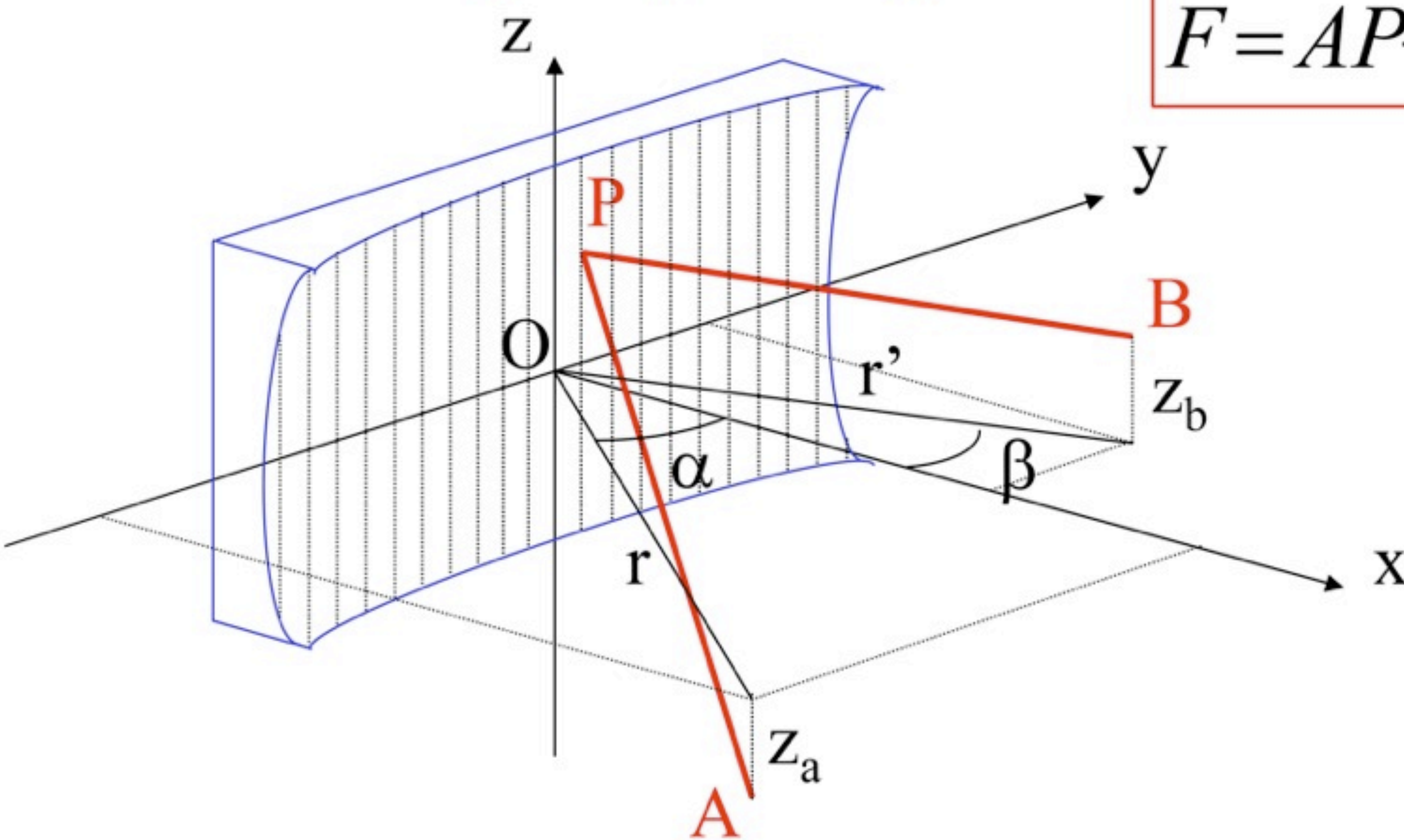
$n\left(\vec{r}\right)$ is the index of refraction of the medium

In other words:
a light-ray going from A to B must traverse an optical path length which is stationary with respect to small variations of that path

Theory of conventional diffraction gratings

For a classical grating with rectilinear grooves parallel to z with constant spacing d, the optical path length is:

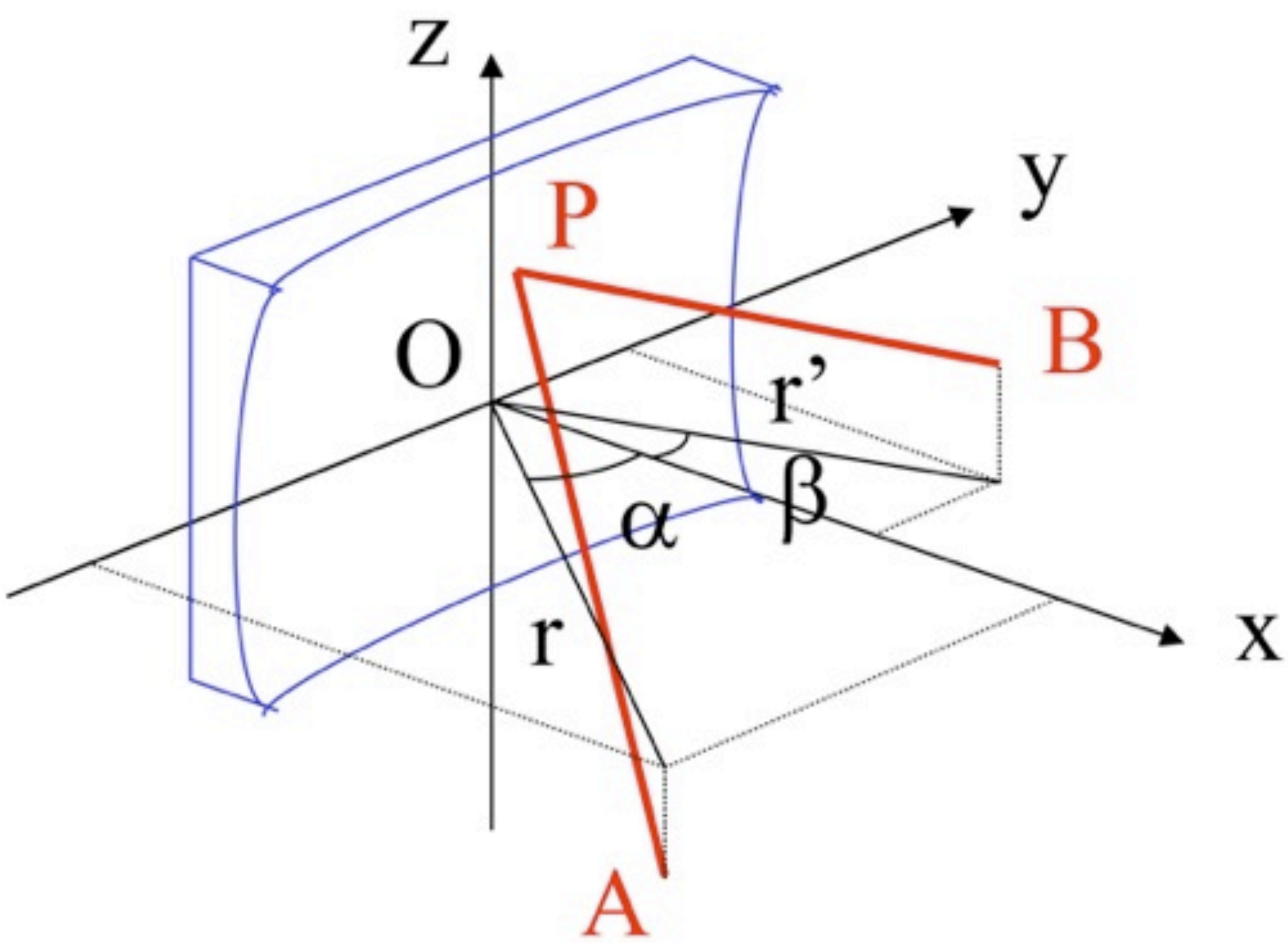
$$F = \overline{AP} + \overline{PB} + kN\lambda y$$



where λ is the wavelength of the diffracted light, k is the order of diffraction (±1,±2,...), N=1/d is the groove density

Perfect focus condition (1)

Let us consider some number of light rays starting from A and impinging on the grating at different points P. Fermat's principle states that if the point A is to be imaged at the point B, then all the optical path lengths from A via the grating surface to B will be the same.



B is the point of a perfect focus if:

$$\frac{\partial F}{\partial y} = 0 \quad \frac{\partial F}{\partial z} = 0$$

for any pair of (y,z)

Perfect focus condition (2)

Equations:

$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

$+$

$$\frac{\partial F}{\partial y} = 0 \quad \frac{\partial F}{\partial z} = 0 \quad \textit{for any pair of } (y,z)$$

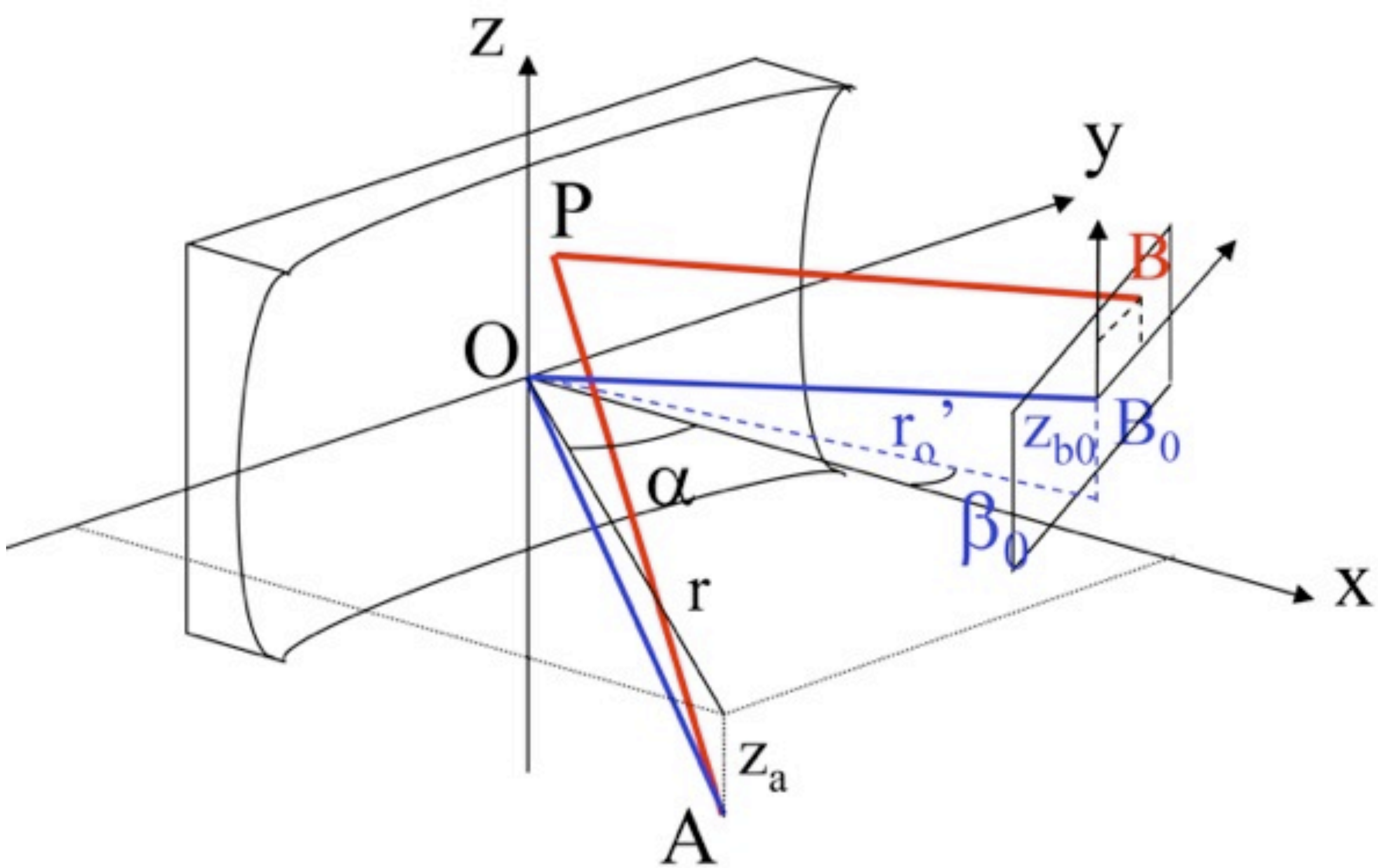
can be used to decide on the required characteristics of the diffraction grating:

- the shape of the surface
- the grooves density
- the object and image distances

Aberrated image

In general, $\frac{\partial F}{\partial y}$ and $\frac{\partial F}{\partial z}$ are functions of y and z and can not be made zero for any y,z

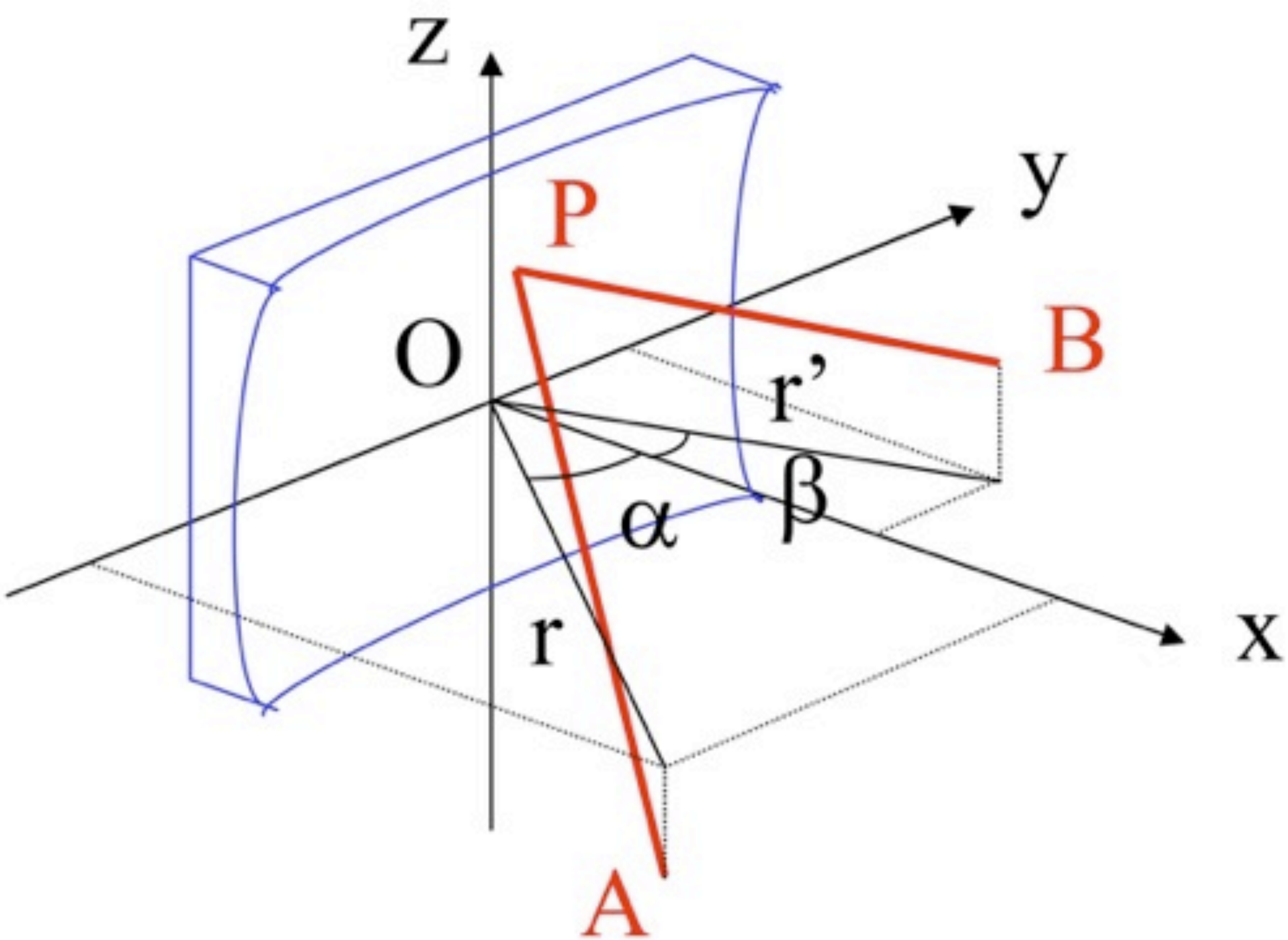
→ when the point P wanders over the grating surface, diffracted rays fall on slightly different points on the focal plane and an aberrated image is formed



- B_0: gaussian image, produced by the central ray
- B: ray diffracted by the generic point P on the grating surface
- Aberrations: displacements of B with respect to B_0

Grating surface

The grating surface may in general be described by a series expansion:



$$x = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} y^i z^j$$

$a_{00} = a_{10} = a_{01} = 0$ because of the choice of origin

j = even if the xy plane is a symmetry plane

Giving suitable values to the coefficients a_{ij}'s we obtain the expressions for the various geometrical surfaces.

a_{ij} coefficients (1)

Toroid

$$a_{02} = \frac{1}{2\rho}; \quad a_{20} = \frac{1}{2R}; \quad a_{22} = \frac{1}{4R^2\rho}; \quad a_{40} = \frac{1}{8R^3};$$

$$a_{04} = \frac{1}{8\rho^3}; \quad a_{12} = 0; \quad a_{30} = 0$$

Sphere, cylinder and plane are special cases of toroid:

R=ρ → **sphere**
R= ∞ → **cylinder**
R=ρ= ∞ → **plane**

Paraboloid

$$a_{02} = \frac{1}{4f\cos\vartheta}; \quad a_{20} = \frac{\cos\vartheta}{4f}; \quad a_{22} = \frac{3\sin^2\vartheta}{32f^3\cos\vartheta};$$

$$a_{12} = -\frac{\tan\vartheta}{8f^2}; \quad a_{30} = -\frac{\sin\vartheta\cos\vartheta}{8f^2}$$

$$a_{40} = \frac{5\sin^2\vartheta\cos\vartheta}{64f^3}; \quad a_{04} = \frac{\sin^2\vartheta}{64f^3\cos^3\vartheta}$$

a_{ij} coefficients (2)

Ellipsoid

$$a_{02} = \frac{1}{4f\cos\vartheta}; \quad a_{20} = \frac{\cos\vartheta}{4f}; \quad a_{04} = \frac{b^2}{64f^3\cos^3\vartheta}\left[\frac{\sin^2\vartheta}{b^2} + \frac{1}{a^2}\right];$$

$$a_{12} = \frac{\tan\vartheta}{8f^2\cos\vartheta}\sqrt{e^2 - \sin^2\vartheta}; \quad a_{30} = \frac{\sin\vartheta}{8f^2}\sqrt{e^2 - \sin^2\vartheta} \;;$$

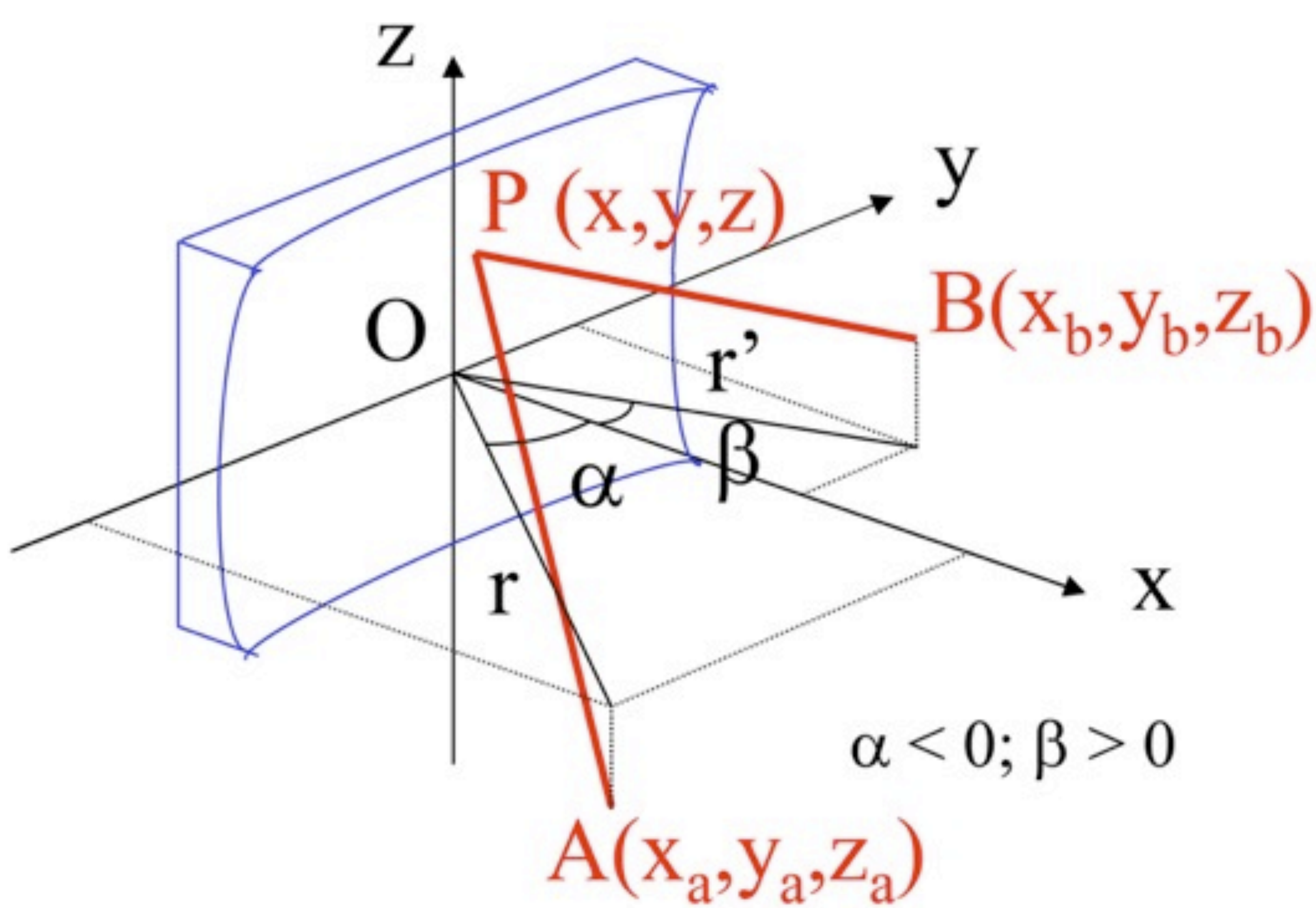
$$a_{40} = \frac{b^2}{64f^3\cos^3\vartheta}\left[\frac{5\sin^2\vartheta \;\; \cos^2\vartheta}{b^2} - \frac{5\sin^2\vartheta}{a^2} + \frac{1}{a^2}\right];$$

$$a_{22} = \frac{\sin^2\vartheta}{16f^3\cos^3\vartheta}\left[\frac{3}{2}\cos^2\vartheta - \frac{b^2}{a^2}\left(1 - \frac{\cos^2\vartheta}{2}\right)\right]$$

$$where \quad f = \left[\frac{1}{r} + \frac{1}{r'}\right]^{-1}$$

http://xdb.lbl.gov/Section4/Sec_4-3Extended.pdf

Optical path function (1)



α < 0; β > 0

$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

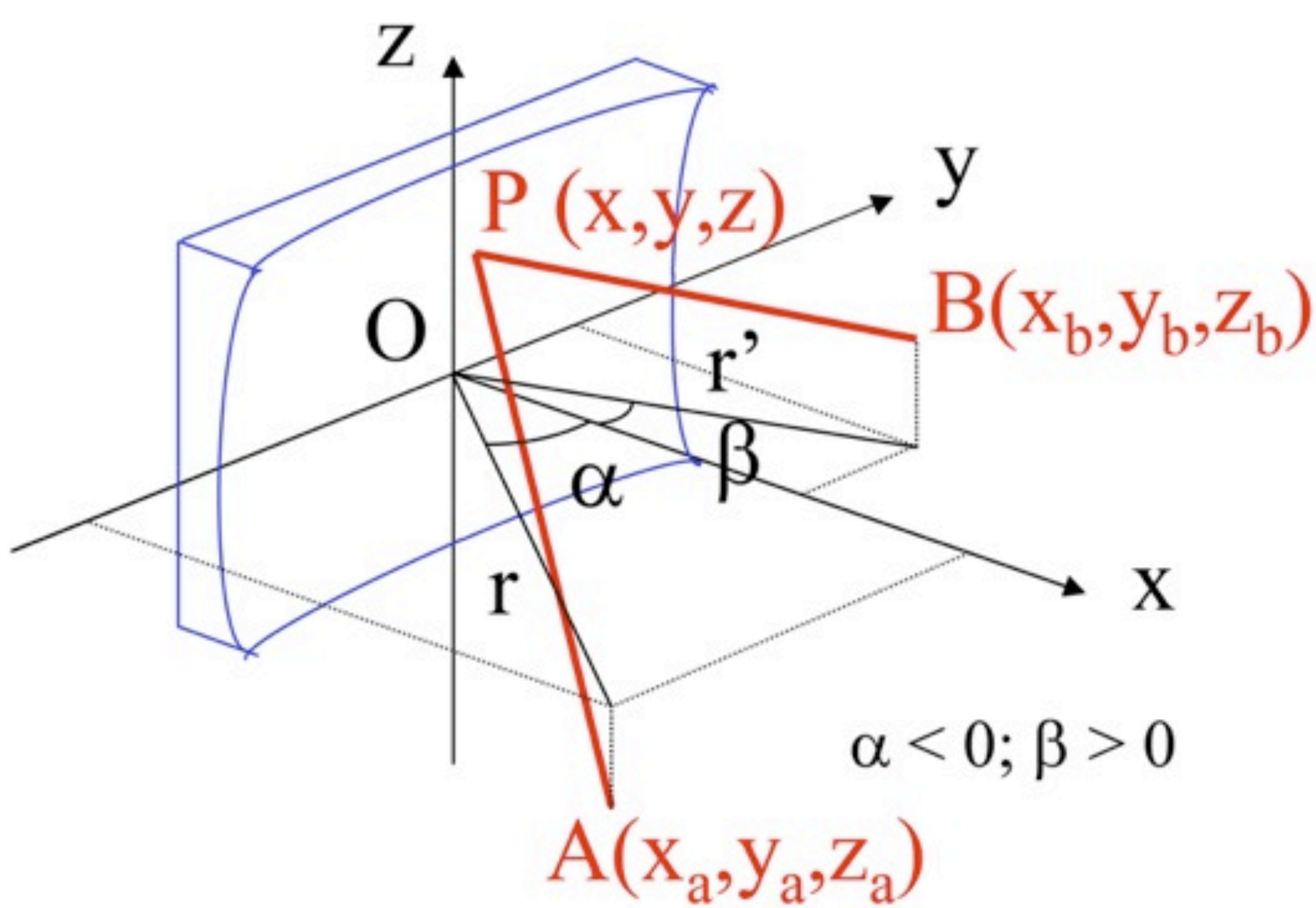
$$\overline{AP} = \sqrt{(x_a - x)^2 + (y_a - y)^2 + (z_a - z)^2}$$

$$\overline{PB} = \sqrt{(x_b - x)^2 + (y_b - y)^2 + (z_b - z)^2}$$

$$x_a = r\cos\alpha \qquad y_a = r\sin\alpha$$

$$x_b = r'\cos\beta \qquad y_b = r'\sin\beta$$

Optical path function (1)



$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

$$\overline{AP} = \sqrt{(x_a - x)^2 + (y_a - y)^2 + (z_a - z)^2}$$

$$\overline{PB} = \sqrt{(x_b - x)^2 + (y_b - y)^2 + (z_b - z)^2}$$

$$x_a = r\cos\alpha \qquad y_a = r\sin\alpha$$

$$x_b = r'\cos\beta \qquad y_b = r'\sin\beta$$

Optical path function (2)

$$F = \sum_{ijk} F_{ijk} y^i z^j$$

$$= F_{000} + yF_{100} + zF_{011} + \frac{1}{2}y^2F_{200} + \frac{1}{2}z^2F_{020} + \frac{1}{2}y^3F_{300}$$

$$+ \frac{1}{2}yz^2F_{120} + \frac{1}{8}y^4F_{400} + \frac{1}{4}y^2z^2F_{220} + \frac{1}{8}z^4F_{040}$$

$$+ yzF_{111} + \frac{1}{2}yF_{102} + \frac{1}{4}y^2F_{202} + \frac{1}{2}y^2zF_{211} + \ldots$$

$$F_{ijk} = z_a^k C_{ijk}(\alpha, r) + z_b^k C_{ijk}(\beta, r') + Nk\lambda f_{ijk}$$

$$f_{ijk} = \begin{cases} 1 & \text{when ijk} = 100 \\ 0 & \text{otherwise} \end{cases}$$

Perfect focus condition (3)

$$\frac{\partial F}{\partial y}=0 \quad \frac{\partial F}{\partial z}=0 \quad \text{for any pair of } (y,z)$$

$$\Downarrow$$

$$F_{ijk}=0 \quad \text{for all } ijk \neq (000)$$

Each term $F_{ijk}y^i z^j$ in the series (except F_{000} and F_{100}) represents a particular type of aberration

F_{ijk} coefficients (1)

for r,r' >> z_a,z_b

$$F_{000} = r + r'$$

$$F_{100} = Nk\lambda - (\sin\alpha + \sin\beta)$$

$$F_{200} = \left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta}{r'}\right) - 2a_{20}(\cos\alpha + \cos\beta)$$

$$F_{020} = \frac{1}{r} + \frac{1}{r'} - 2a_{02}(\cos\alpha + \cos\beta)$$

$$F_{300} = \left[\frac{T(r,\alpha)}{r}\right]\sin\alpha + \left[\frac{T(r',\beta)}{r'}\right]\sin\beta - 2a_{30}(\cos\alpha + \cos\beta)$$

$$F_{120} = \left[\frac{S(r,\alpha)}{r}\right]\sin\alpha + \left[\frac{S(r',\beta)}{r'}\right]\sin\beta - 2a_{12}(\cos\alpha + \cos\beta)$$

where $T(r,\alpha) = \frac{\cos^2\alpha}{r} - 2a_{20}\cos\alpha$ and $S(r,\alpha) = \frac{1}{r} - 2a_{02}\cos\alpha$

and analogous expressions for $T(r',\beta)$ and $S(r',\beta)$

F_{ijk} coefficients (2)

$$F_{400} = \left[\frac{4T(r,\alpha)}{r^2}\right]\sin^2\alpha - \left[\frac{T^2(r,\alpha)}{r}\right] + \left[\frac{4T(r',\beta)}{r'^2}\right]\sin^2\beta - \left[\frac{T^2(r',\beta)}{r'}\right]$$

$$-8a_{30}\left[\frac{\sin\alpha\cos\alpha}{r} + \frac{\sin\beta\cos\beta}{r'}\right] - 8a_{40}(\cos\alpha + \cos\beta) + 4a_{20}^2\left[\frac{1}{r} + \frac{1}{r'}\right]$$

$$F_{220} = \left[\frac{2S(r,\alpha)}{r^2}\right]\sin^2\alpha + \left[\frac{2S(r',\beta)}{r'^2}\right]\sin^2\beta - \left[\frac{T(r,\alpha)S(r,\alpha)}{r}\right] - \left[\frac{T(r',\beta)S(r',\beta)}{r'}\right]$$

$$+4a_{20}a_{02}\left[\frac{1}{r} + \frac{1}{r'}\right] - 4a_{22}(\cos\alpha + \cos\beta) - 4a_{12}\left[\frac{\sin\alpha\cos\alpha}{r} + \frac{\sin\beta\cos\beta}{r'}\right]$$

$$F_{040} = 4a_{02}^2\left[\frac{1}{r} + \frac{1}{r'}\right] - 8a_{04}(\cos\alpha + \cos\beta) - \left[\frac{S^2(r,\alpha)}{r}\right] - \left[\frac{S^2(r',\beta)}{r'}\right]$$

F_{ijk} coefficients (3)

$$F_{011} = -\frac{z_a}{r} - \frac{z_b}{r'}$$

$$F_{111} = -\frac{z_a \sin\alpha}{r^2} - \frac{z_b \sin\beta}{r'^2}$$

$$F_{102} = \frac{{z_a}^2 \sin\alpha}{r^2} + \frac{{z_b}^2 \sin\beta}{r'^2}$$

$$F_{202} = \left(\frac{z_a}{r}\right)^2 \left[\frac{2\sin^2\alpha}{r} - T(r,\alpha)\right] + \left(\frac{z_b}{r'}\right)^2 \left[\frac{2\sin^2\beta}{r'} - T(r',\beta)\right]$$

$$F_{211} = \frac{z_a}{r^2}\left[T(r,\alpha) - \frac{2\sin^2\alpha}{r}\right] + \frac{z_b}{r'^2}\left[T(r',\beta) - \frac{2\sin^2\beta}{r'}\right]$$

Gaussian image point (1)

If we apply Fermat's principle to the central ray: $\left(\frac{\partial F}{\partial y}\right)_{y=0,z=0} = 0 \quad \left(\frac{\partial F}{\partial z}\right)_{y=0,z=0} = 0$

$F_{100} = 0$ ⟹ $\sin\alpha + \sin\beta_0 = Nk\lambda$ **grating equation**

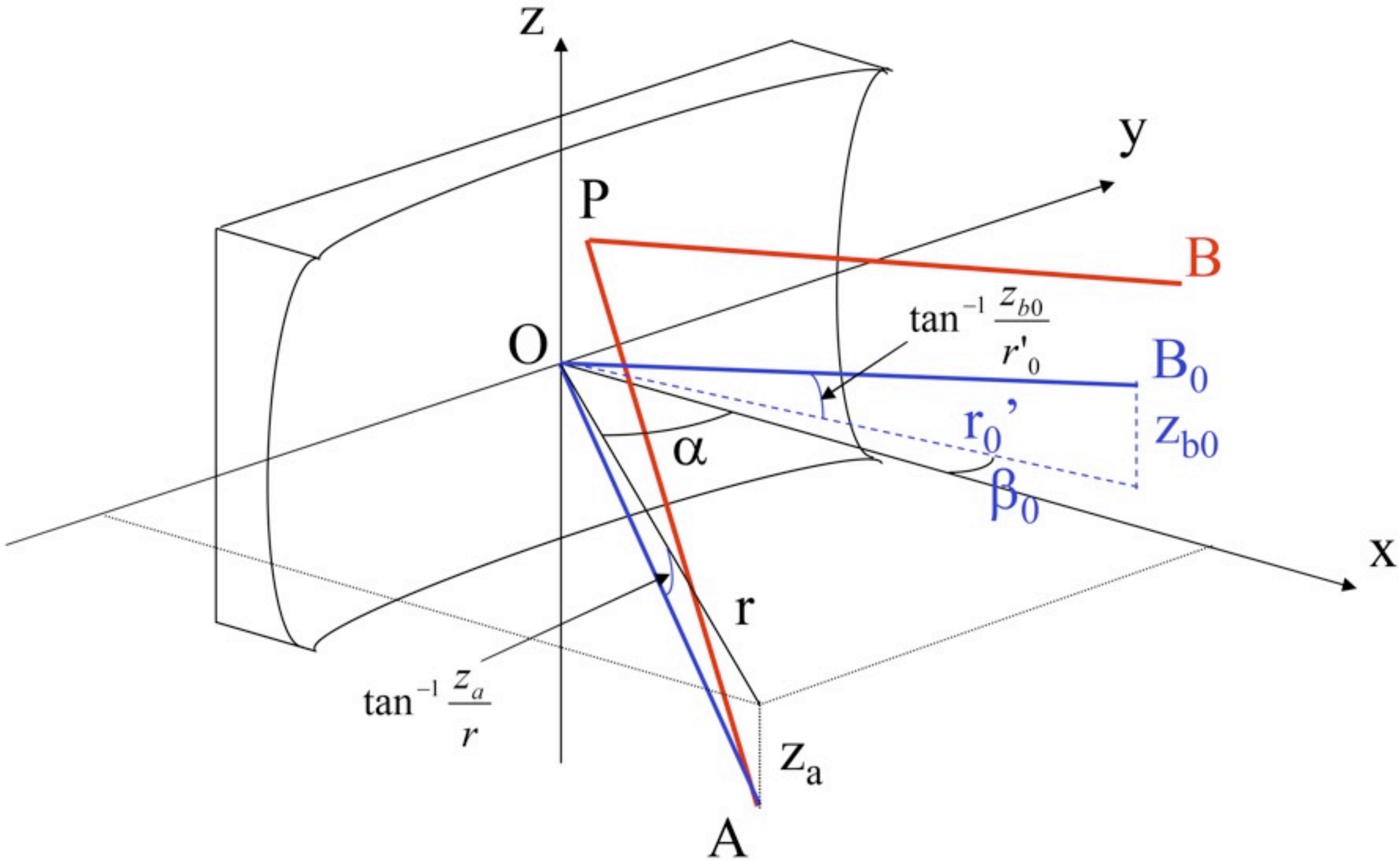
$F_{011} = 0$ ⟹ $\frac{z_a}{r} = -\frac{z_{b0}}{r'_0}$ law of magnification in the sagittal direction

The tangential focal distance r'_0 is obtained by setting:

$F_{200} = 0$ ⟹ $\left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta_0}{r'_0}\right) - 2a_{20}(\cos\alpha + \cos\beta_0) = 0$ **tangential focusing**

The three above equations determine the Gaussian image point $B_0(r'_0, \beta_0, z_{b0})$

Gaussian image point (2)



Sagittal focusing

While the second order aberration term F_{200} governs the tangential focusing, the second order term F_{020} governs the sagittal focusing:

$$F_{020} = 0 \quad \Longrightarrow \quad \frac{1}{r} + \frac{1}{r'} - 2a_{02}(\cos\alpha + \cos\beta) = 0$$

sagittal focusing

Example: toroidal mirror

Substituting $a_{02} = \dfrac{1}{2\rho}$; $a_{20} = \dfrac{1}{2R}$ in $F_{200} = 0$; $F_{020} = 0$

and imposing $\alpha = -\beta = \theta$

$$\Longrightarrow \quad \left(\frac{1}{r} + \frac{1}{r_t'}\right)\frac{\cos\vartheta}{2} = \frac{1}{R} \qquad \left(\frac{1}{r} + \frac{1}{r_s'}\right)\frac{1}{2\cos\vartheta} = \frac{1}{\rho}$$

Sagittal focusing

While the second order aberration term F_{200} governs the tangential focusing, the second order term F_{020} governs the sagittal focusing:

$$F_{020} = 0 \quad \Longrightarrow \quad \frac{1}{r} + \frac{1}{r'} - 2a_{02}(\cos\alpha + \cos\beta) = 0$$

sagittal focusing

Example: toroidal mirror

Substituting $a_{02} = \frac{1}{2\rho}$; $a_{20} = \frac{1}{2R}$ in $F_{200} = 0$; $F_{020} = 0$

and imposing $\alpha = -\beta = \theta$

$$\Longrightarrow \quad \left(\frac{1}{r} + \frac{1}{r_t'}\right)\frac{\cos\vartheta}{2} = \frac{1}{R} \qquad \left(\frac{1}{r} + \frac{1}{r_s'}\right)\frac{1}{2\cos\vartheta} = \frac{1}{\rho}$$

Aberrations terms

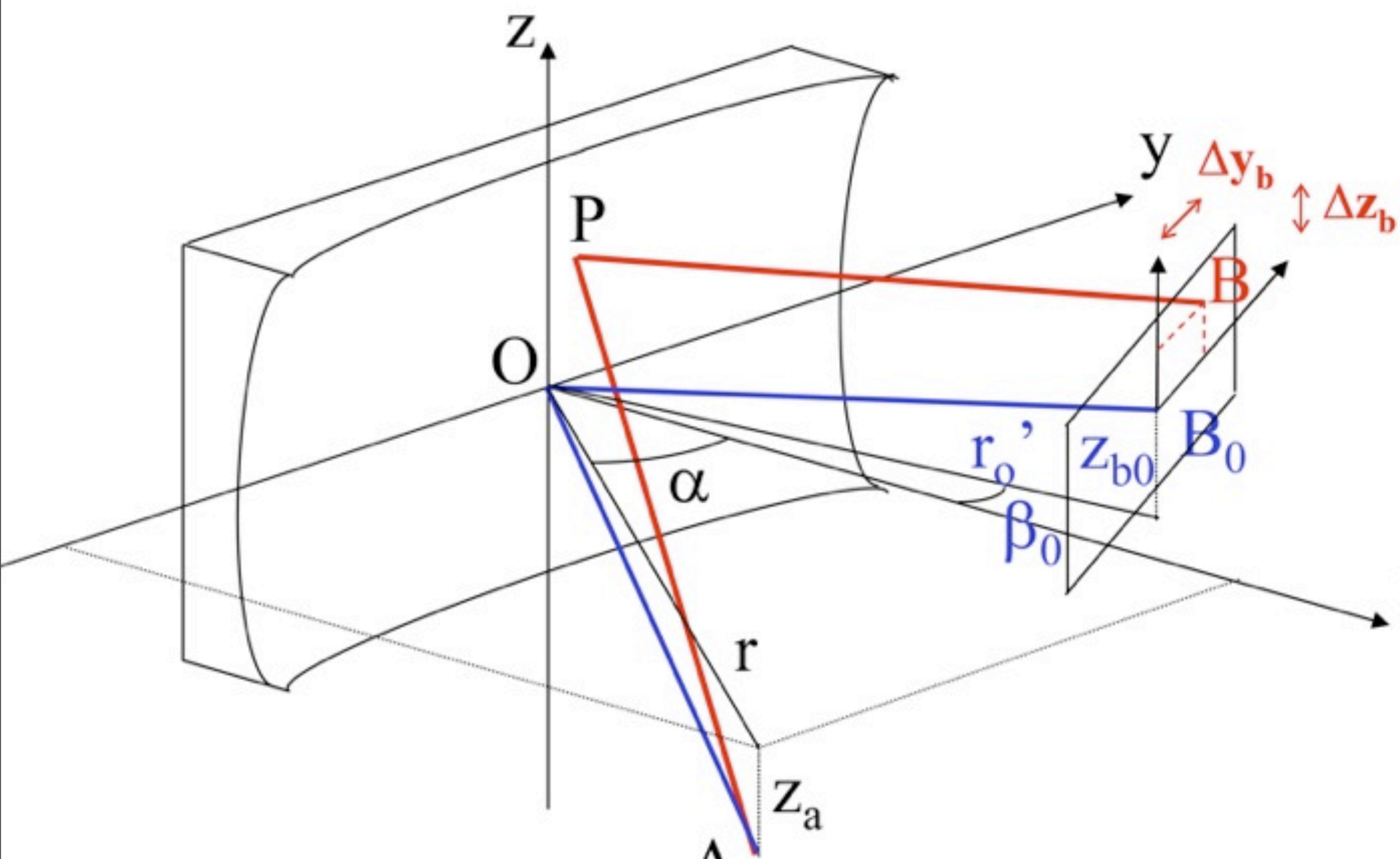
Most important imaging errors:

F_{200}	defocus
F_{020}	astigmatism
F_{300}	primary coma (aperture defect)
F_{120}	astigmatic coma
F_{400} F_{220} F_{040}	spherical aberration

There is an ambiguity in the naming of the aberrations in the grazing incidence case!

Ray aberrations (1)

The generic ray starting from A will arrive at the focal plane at a point B displaced from the Gaussian image point B_0 by the ray aberrations Δy_b and Δz_b:



$$\Delta y_b = \frac{r_0'}{\cos\beta_0}\frac{\partial F}{\partial y}$$

$$\Delta z_b = r_0' \frac{\partial F}{\partial z}$$

Ray aberrations (2)

Substituting the expansion of F , the ray aberrations for each aberration type can be calculated separately:

$$\Delta y_b{}^{ijk} = \frac{r_0'}{\cos\beta_0} F_{ijk}\ i\, y^{i-1}\, z^j$$

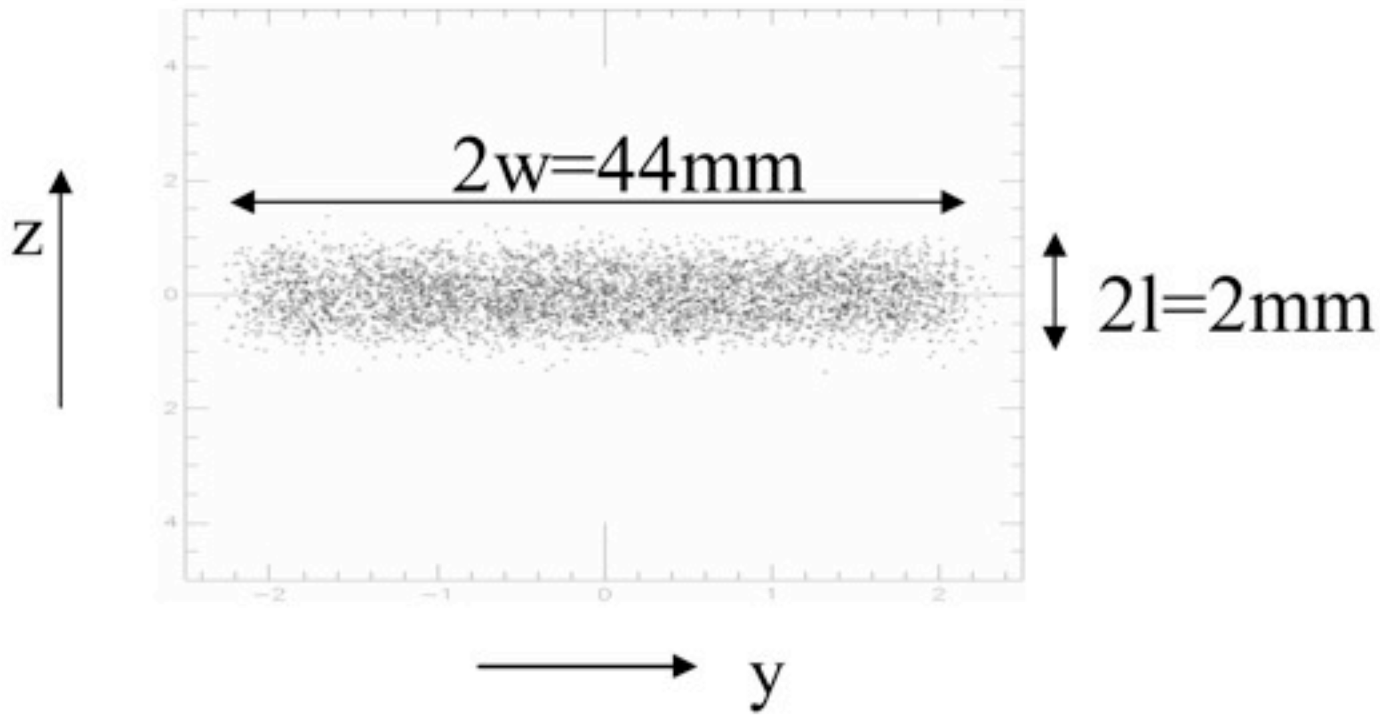
$$\Delta z_b{}^{ijk} = r_0'\, F_{ijk}\ y^i\, j\, z^{j-1}$$

Provided the aberrations are not too large, they are additive: they may either reinforce or cancel.

$$\Delta y_b = \sum_{ijk} \Delta y_b{}^{ijk} \qquad \Delta z_b = \sum_{ijk} \Delta z_b{}^{ijk}$$

Aberrated image

Example of footprint on the grating:



Substituting y=±w and z=±l in the ray aberrations Δy_b^{ijk} and Δz_b^{ijk} , we evaluate the contributions of the rays which are more distant from the pole of the grating

→ size ($\Delta y_b * \Delta z_b$) of the resulting aberrated image

Defocus and coma contributions

The defocus contribution is linear in the ruled length (± w) of the grating, the error in the dispersive direction is symmetric about the Gaussian image point:

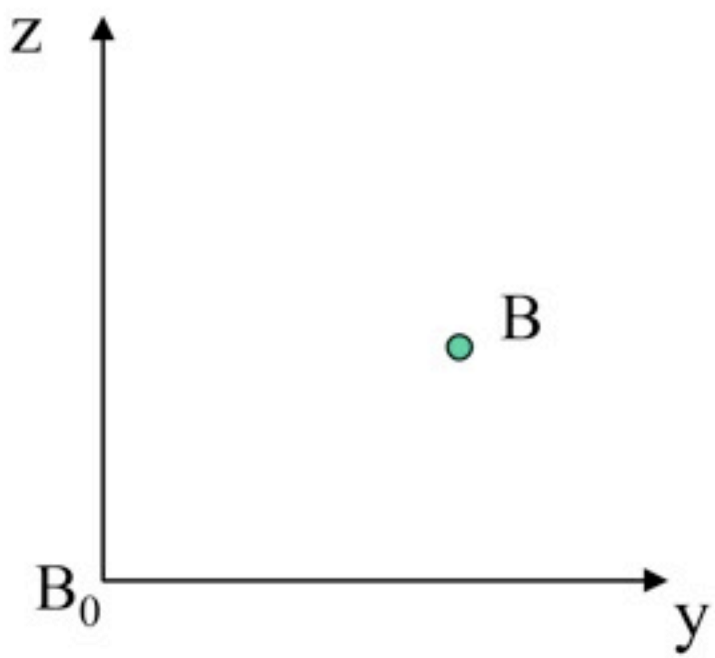
$$\Delta y_b{}^{200}(\pm w) = \pm \frac{r_0'}{\cos\beta_0} F_{200}\, 2\, w$$

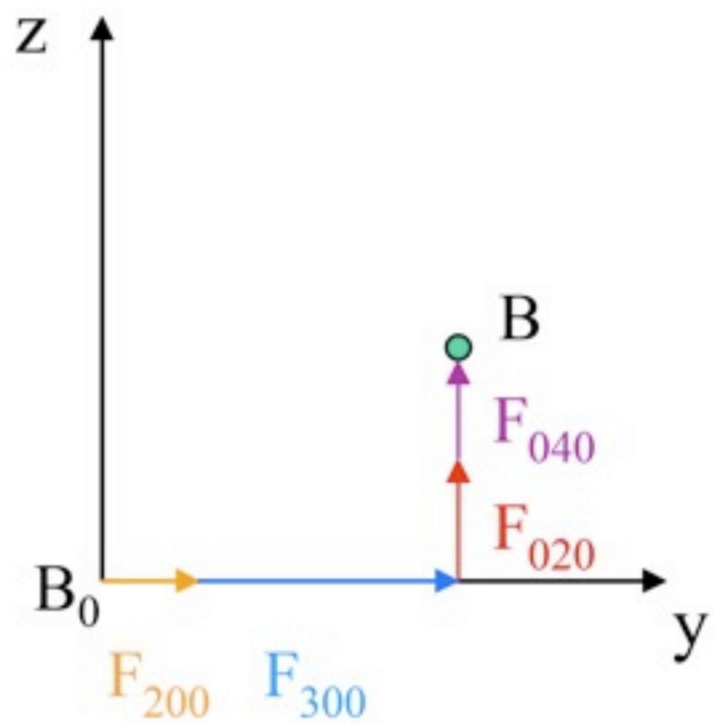
The coma contribution is proportional to w^2 giving a dispersive error which only occurs on one side of the Gaussian image point for rays from both the top and the bottom of the grating (y=±w):

$$\Delta y_b{}^{300}(\pm w) = \frac{r_0'}{\cos\beta_0} F_{300}\, 3\, w^2$$

Comparison ray trace - aberration calculations

Example





Ray trace simple tells us that the ray arrives in a certain point

Aberration-based calculations specify the different contributions

Aberrations contribution to resolution

$$\Delta\lambda = \left(\frac{\partial\lambda}{\partial\beta}\right)_{\alpha=const} \Delta\beta$$

$$= \frac{\cos\beta}{Nk}\Delta\beta$$

Substituting: $\Delta\beta = \frac{\Delta y_b}{r'}$ → $\Delta\lambda = \frac{\cos\beta}{Nk}\frac{\Delta y_b}{r'}$

Substituting: $\Delta y_b = \frac{r_0'}{\cos\beta_0}\frac{\partial F}{\partial y}$ → $\Delta\lambda = \frac{1}{Nk}\frac{\partial F}{\partial y}$

$$\boxed{\Delta\lambda = \frac{1}{Nk}\sum_{ijk} F_{ijk}\; i\, y^{i-1}\, z^{j}}$$

Aberration theory: conclusions

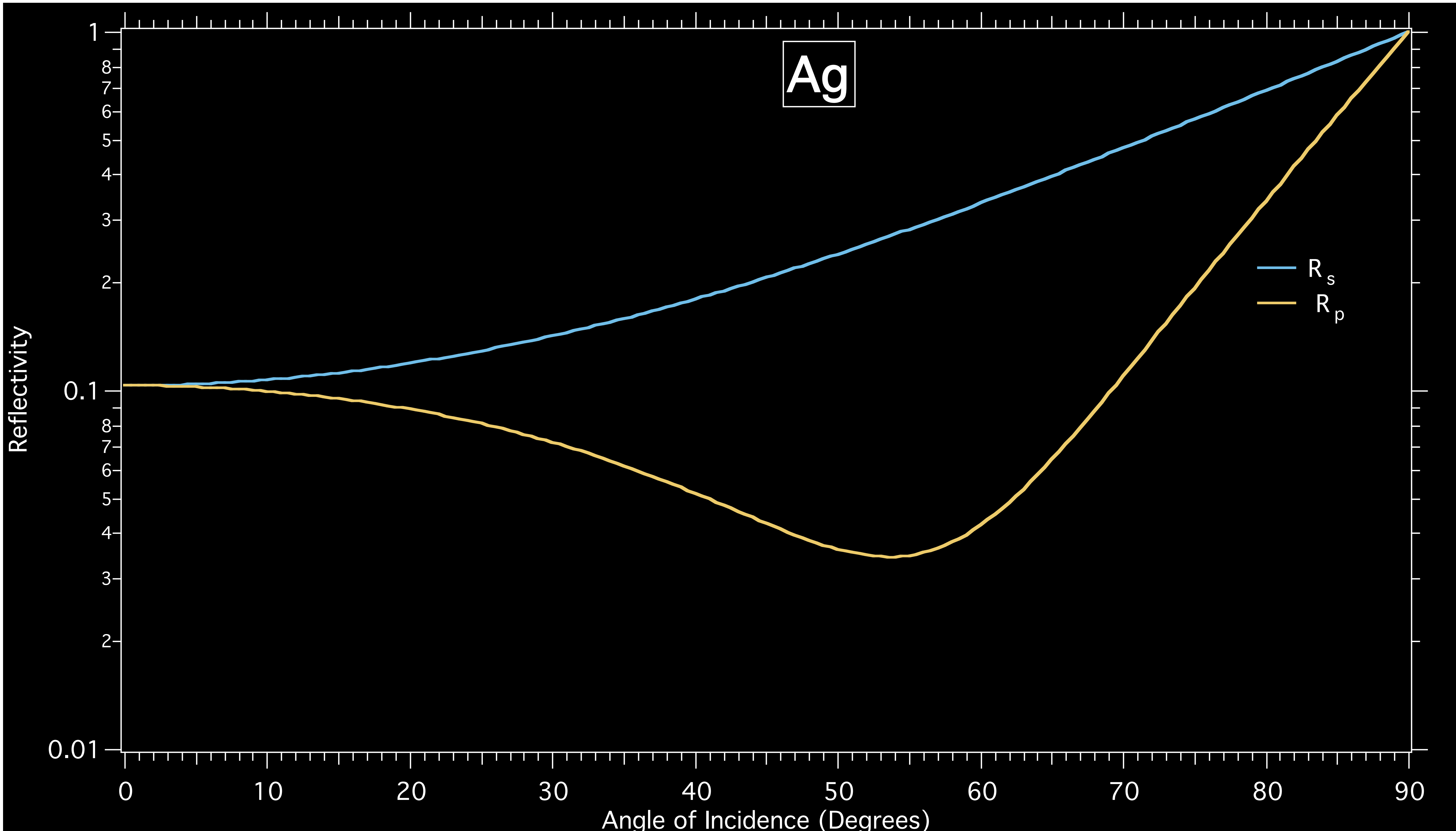
• Perfect focus condition: $\frac{\partial F}{\partial y} = 0 \quad \frac{\partial F}{\partial z} = 0$ for each pair (y,z)
→ all the coefficients F_{ijk} must be zero

• Non-zero values for the coefficients F_{ijk} lead to displacements of the rays arriving in the image plane from the ideal Gaussian image point.

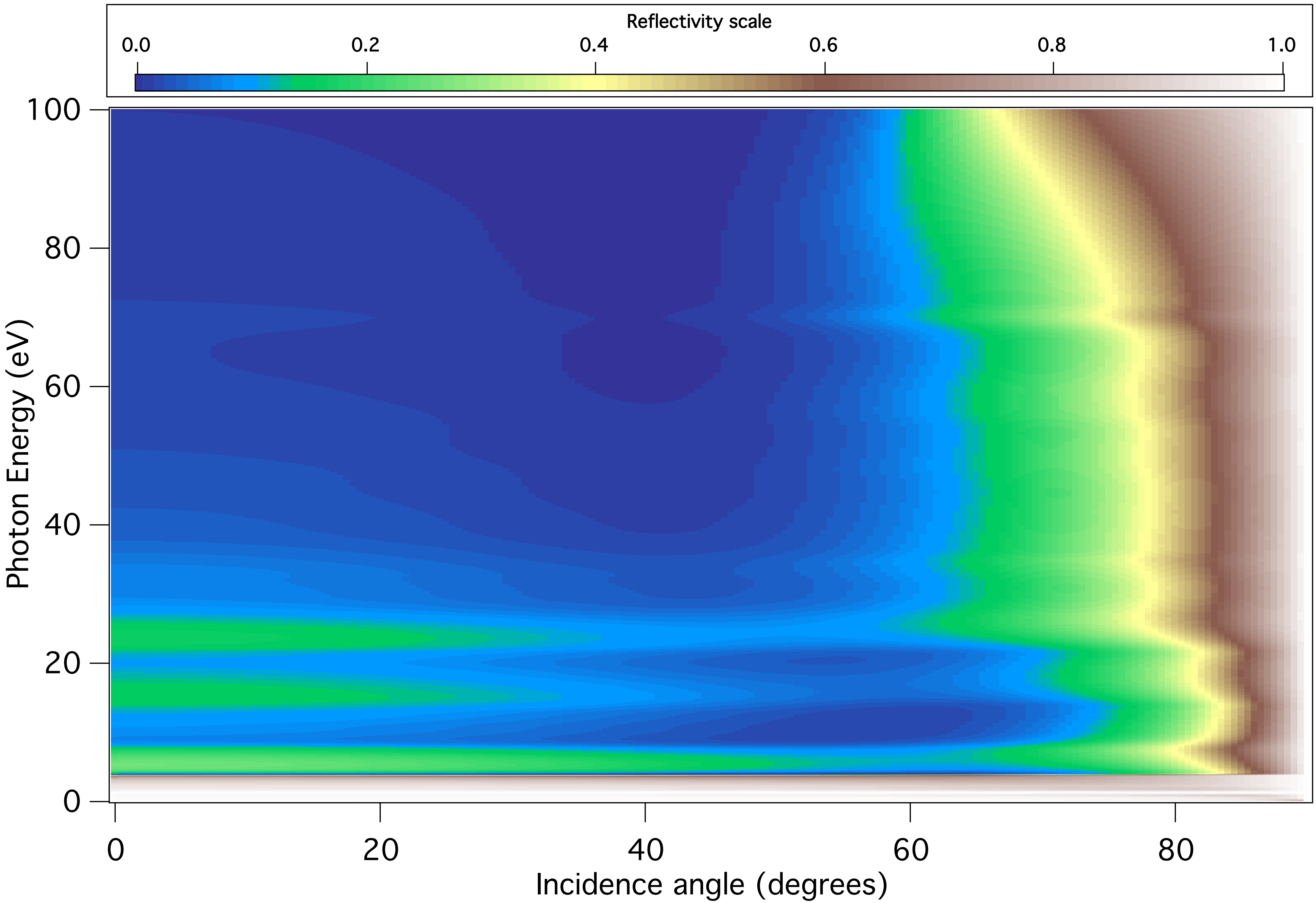
• We have found the expressions for these rays displacements and the corresponding contributions to wavelength resolution. In this way the impact on the imaging and energy resolution properties of a given grating can be evaluated.

• By a proper choice of the grating shape, groove density, object and image distances, the sum of the aberrations may be reduced to a minimum.

Ag s- and p- polarized reflectivity at $\hbar\omega$=20eV



Experimental reflectivity of Ag for p-polarized light



Refraction index

In the Lorentz-Drude model we can write the refraction index as

$$n(\omega) = \left[1 - \frac{e^2 n_a}{\epsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i\gamma}\right]^{\frac{1}{2}}$$

where the ω_s can be regarded as the absorption edges of the atoms constituting the medium. At high photon energies, the expression becomes:

$$n(\omega) \simeq 1 - \frac{1}{2} \frac{e^2 n_a}{\epsilon_0 m} \sum_s \frac{g_s}{(\omega^2 - \omega_s^2) + i\gamma}$$

Refraction index

so we can write the refraction index as

$$n \approx 1 - \delta + i\beta$$

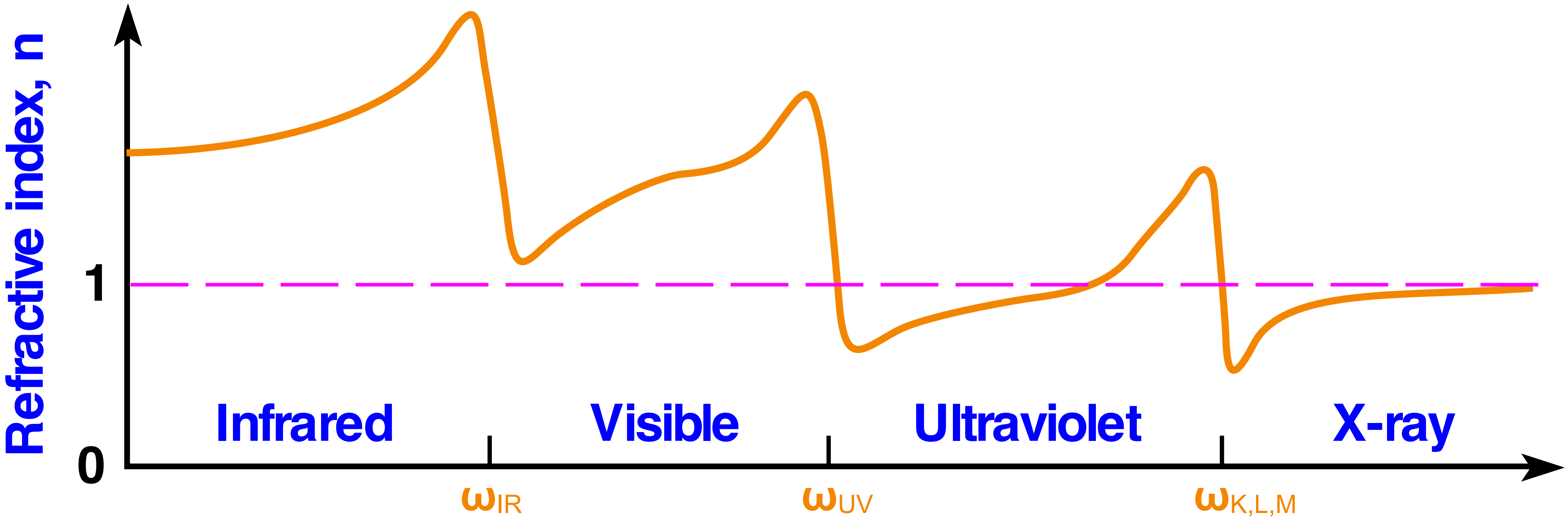
where, if we assume a single absorption edge at λ_s:

$$\delta \approx \frac{e^2\lambda^2}{2\pi mc^2}\left|N + N_s g_s \left(\frac{\lambda}{\lambda_s}\right)^2 \ln\left[\left(\frac{\lambda_s}{\lambda}\right)^2 - 1\right]\right|$$

and for $\lambda << \lambda_s$ (free electron limit):

$$\delta \approx \frac{Ne^2\lambda^2}{2\pi mc^2}$$

Refraction index



λ^2 behaviour

λ & δ ≪ 1

δ crossover

F30 effect (primary coma)

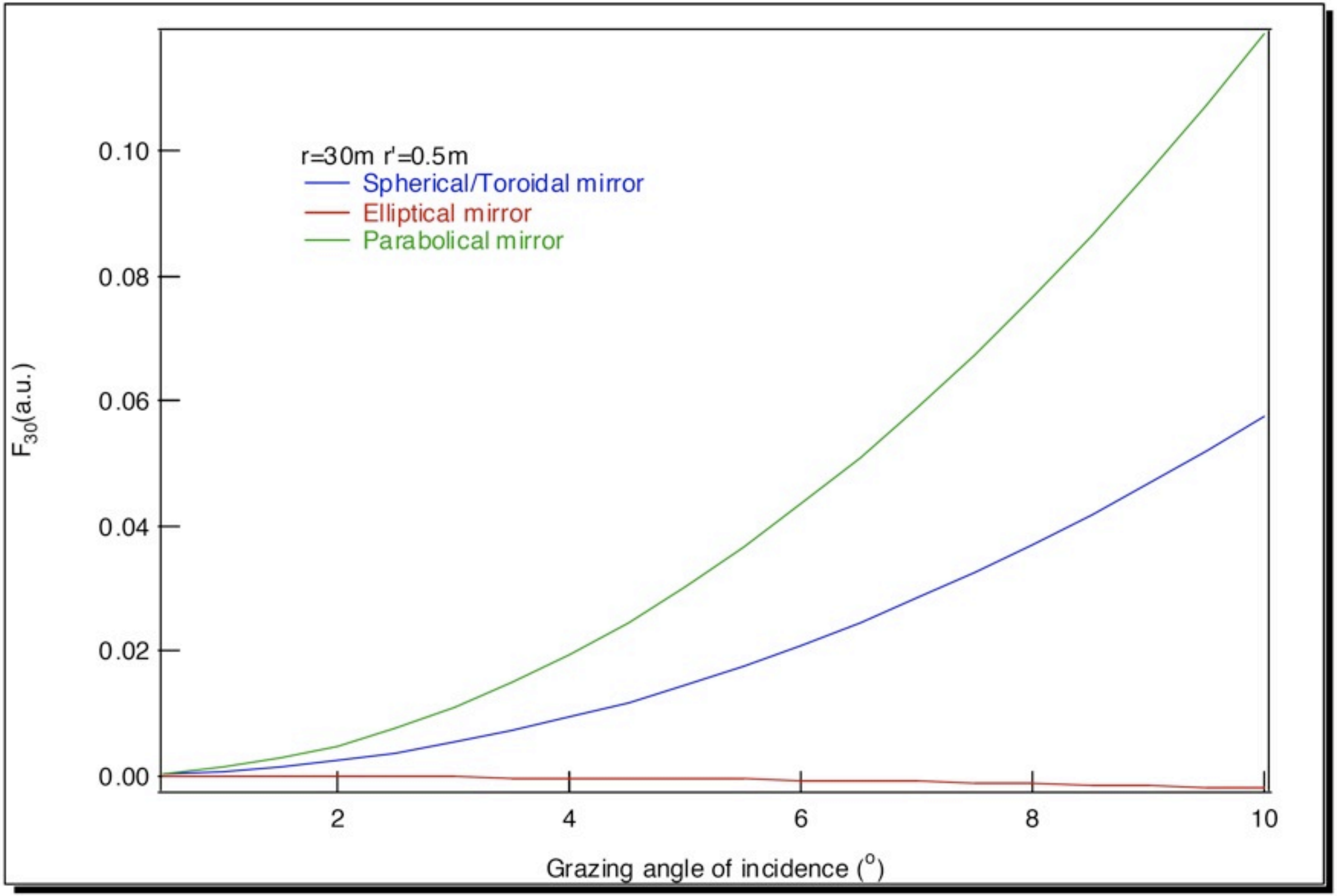
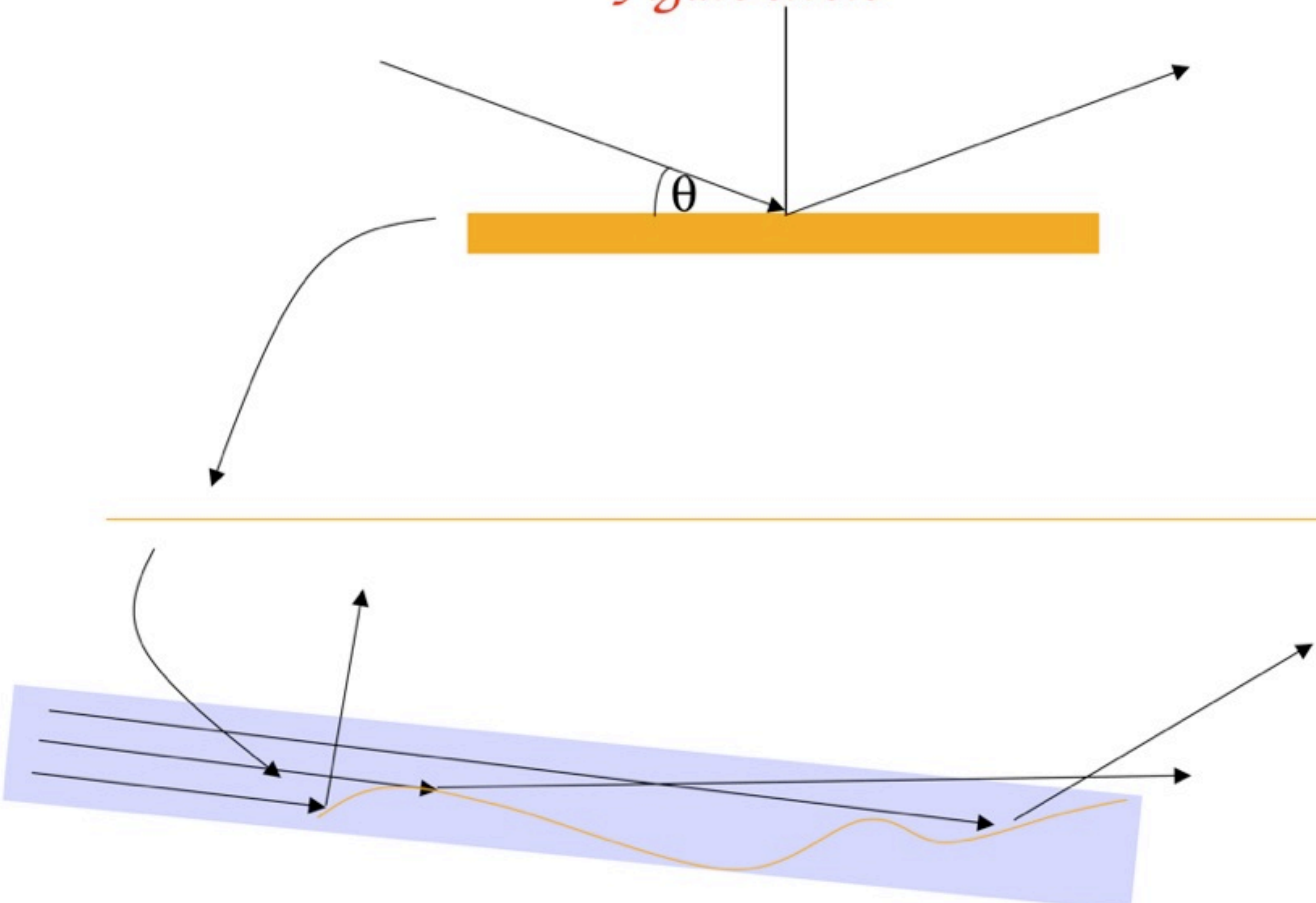
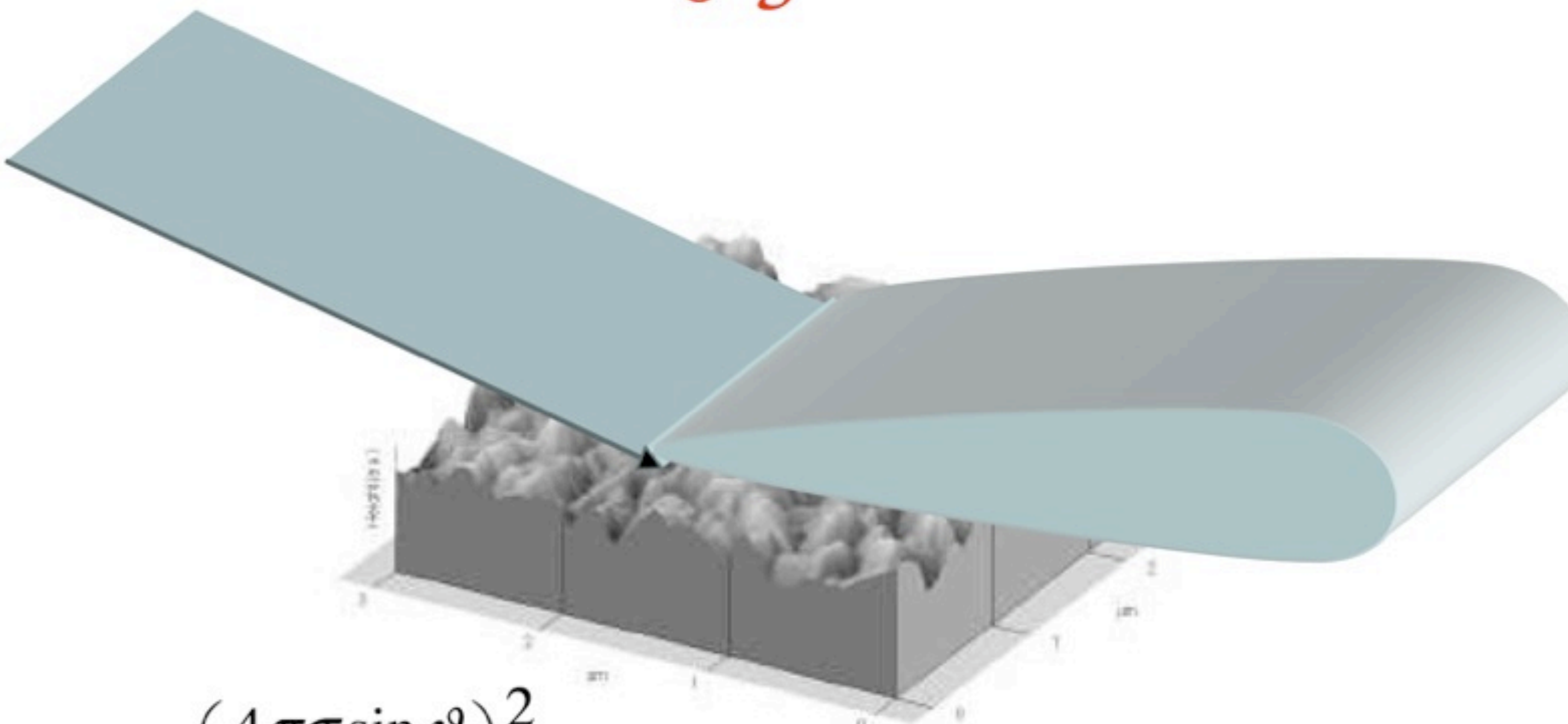


Figure errors



Roughness



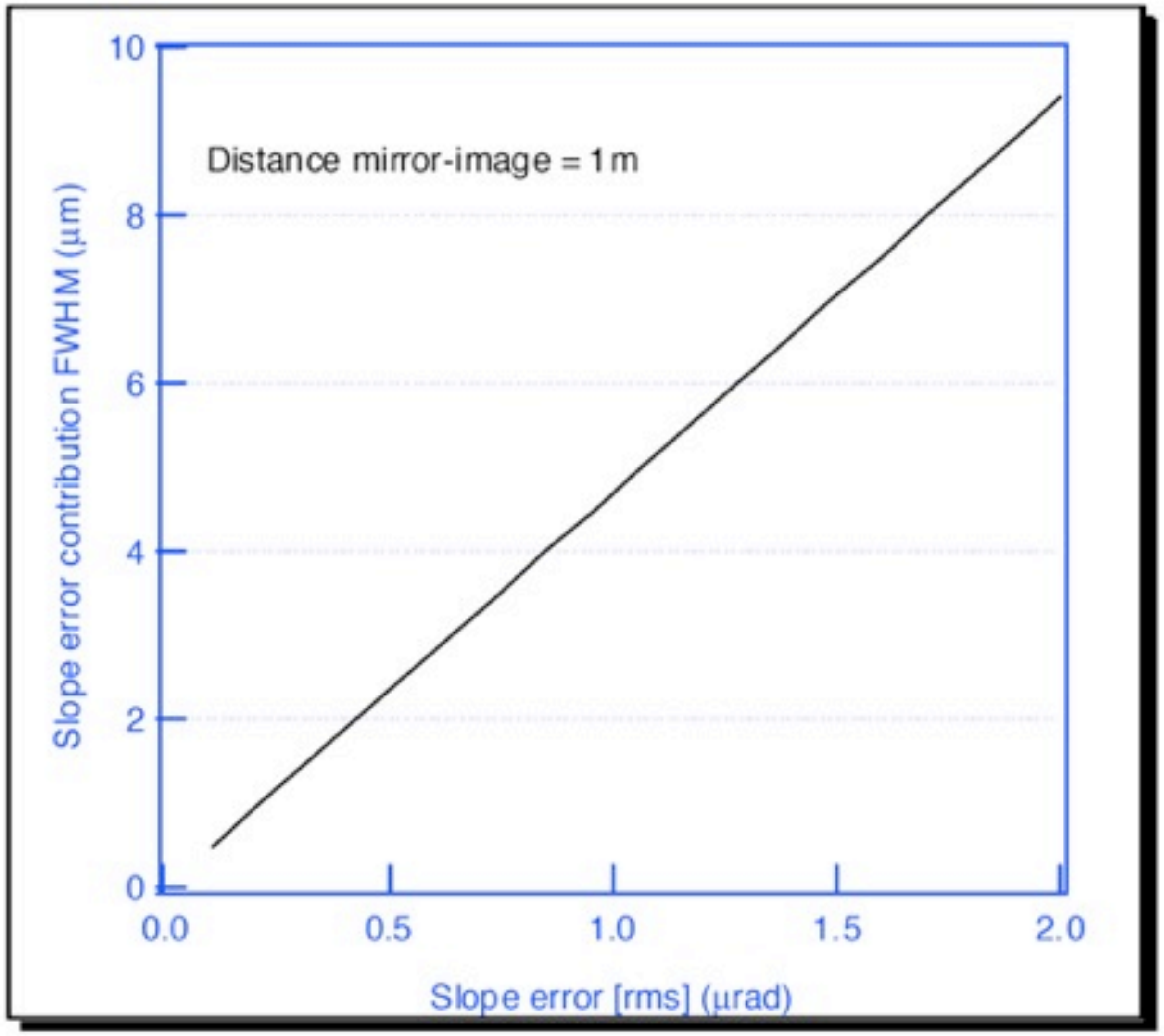
$$I = I_0 e^{-\left(\frac{4\pi\sigma\sin\vartheta}{\lambda}\right)^2}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{x=0}^{n}\left[s(x) - \overline{s(x)}\right]^2}$$

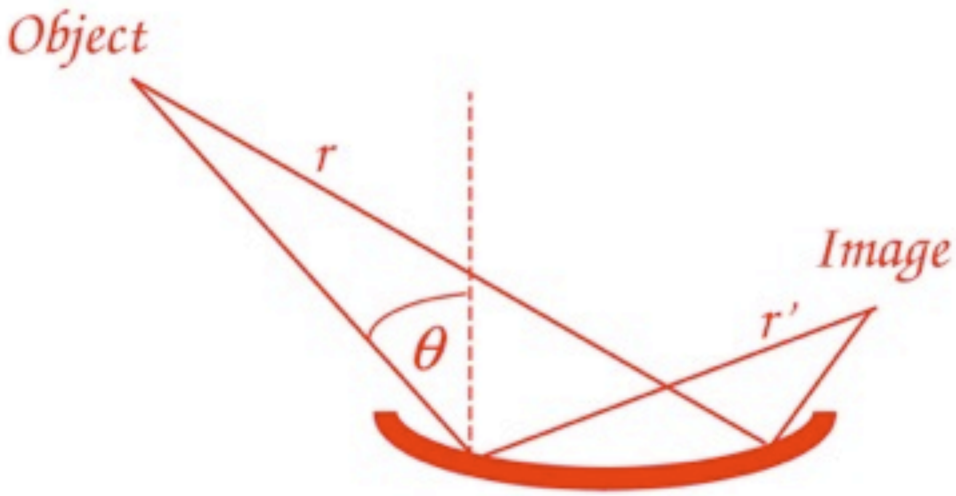
Slope errors (tangential)

Typical manufacturer capabilities (SESO, ZEISS, Winlight, Jobin Yvon)

Shape	Lenght	rms errors
Spherical/flat	Up to 500 mm	< 0.5 µrad
Spherical/flat	> 500 mm	1-2 µrad
Toroidal	Up to 500 mm	< 1 µrad
Toroidal	> 500 mm	> 1 µrad
Aspherical	Up to 500 mm	2 µrad
Aspherical	> 500 mm	3-5 µrad



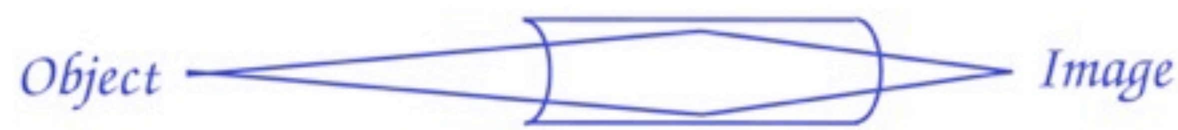
Focal property



Tangential focusing

Term $\mathcal{F}_{20}$ of the optical path function

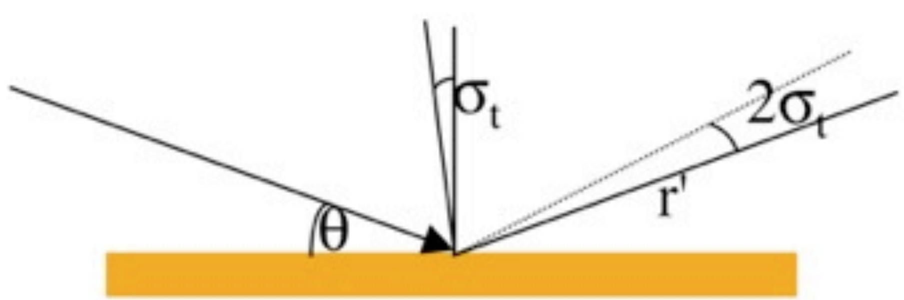
$(1/r+1/r')\cos\theta/2=1/\mathcal{R}$ spherical mirror



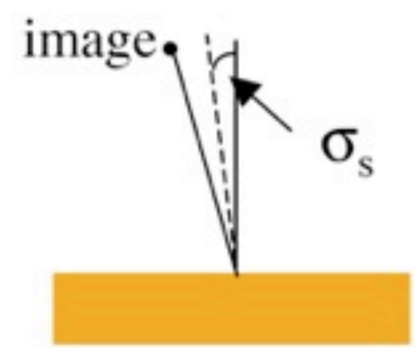
Sagital focusing

Term $\mathcal{F}_{02}$ of the optical path function

$(1/r+1/r')/(2\cos\theta)=1/\mathcal{R}$ cylindrical/toroidal mirror

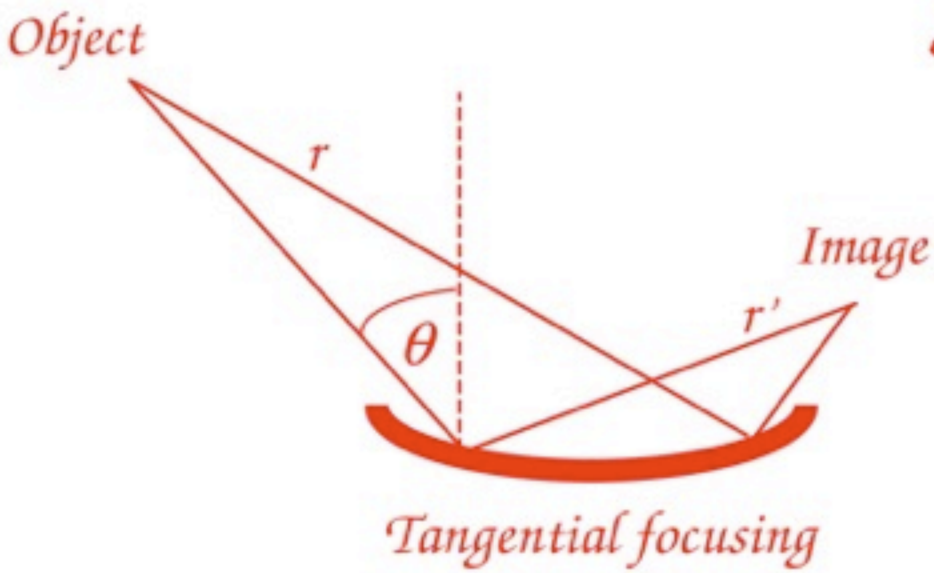


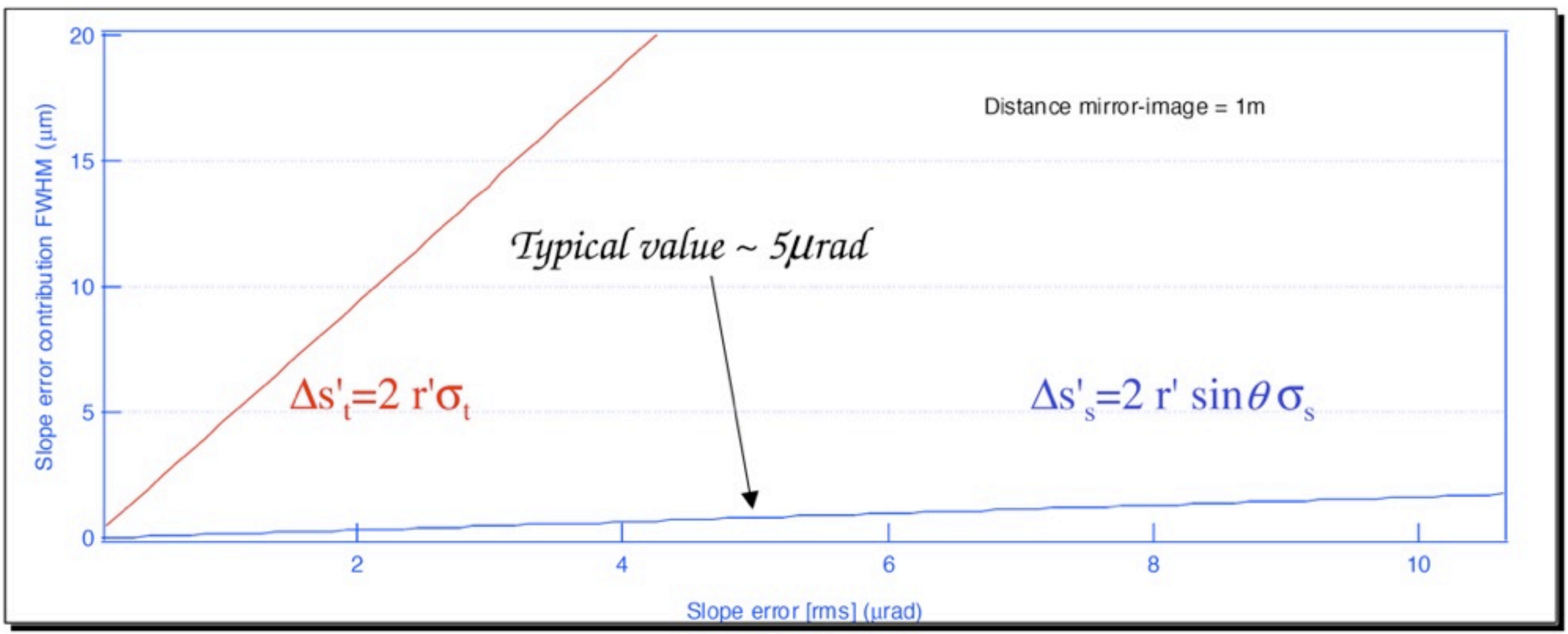
$\Delta s'_t=2\ r'\sigma_t$



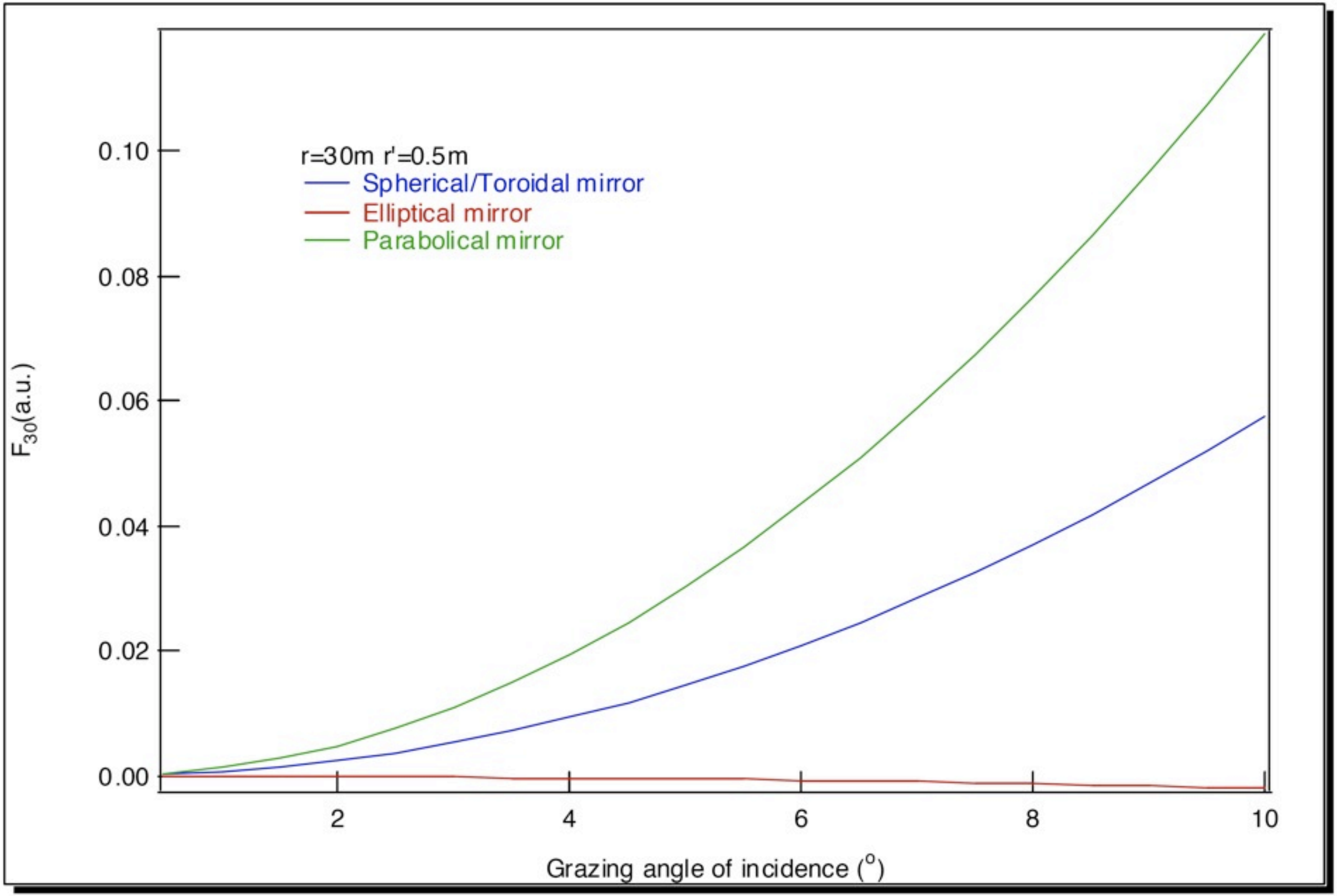
$\Delta s'_s=2\ r'\ \sin\theta\,\sigma_s$

Slope error effect





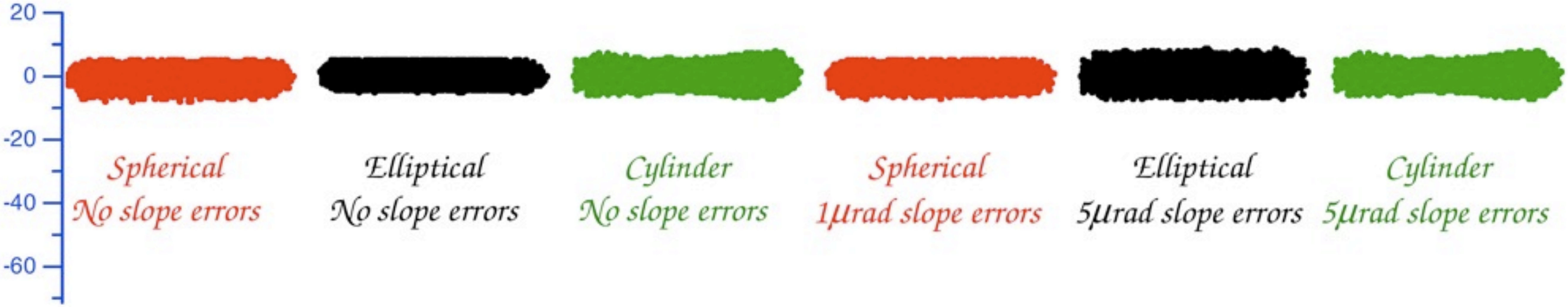
F30 effect (primary coma)



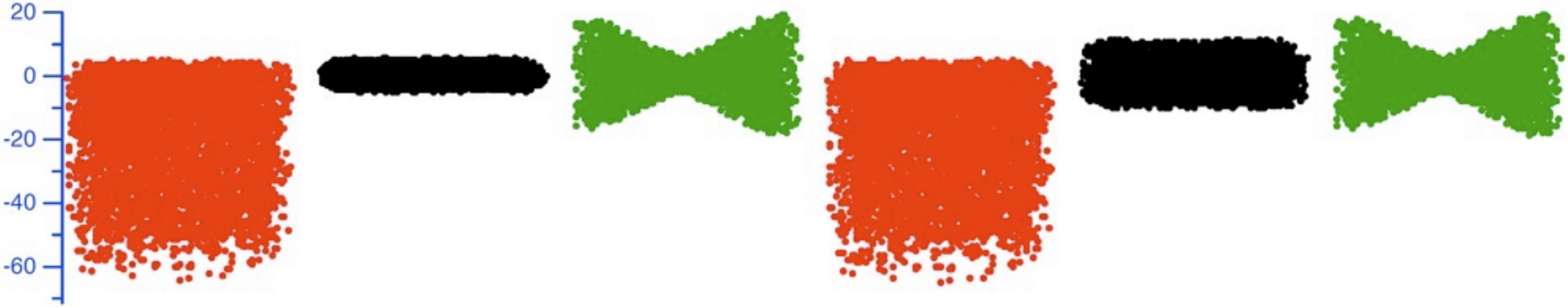
Final focus (F30-F03)

source 80 μm vertical; r=4000 mm r´=400 mm (10:1) θ=88°

Beam divergence 100X100 μrad



Beam divergence 500X500 μrad

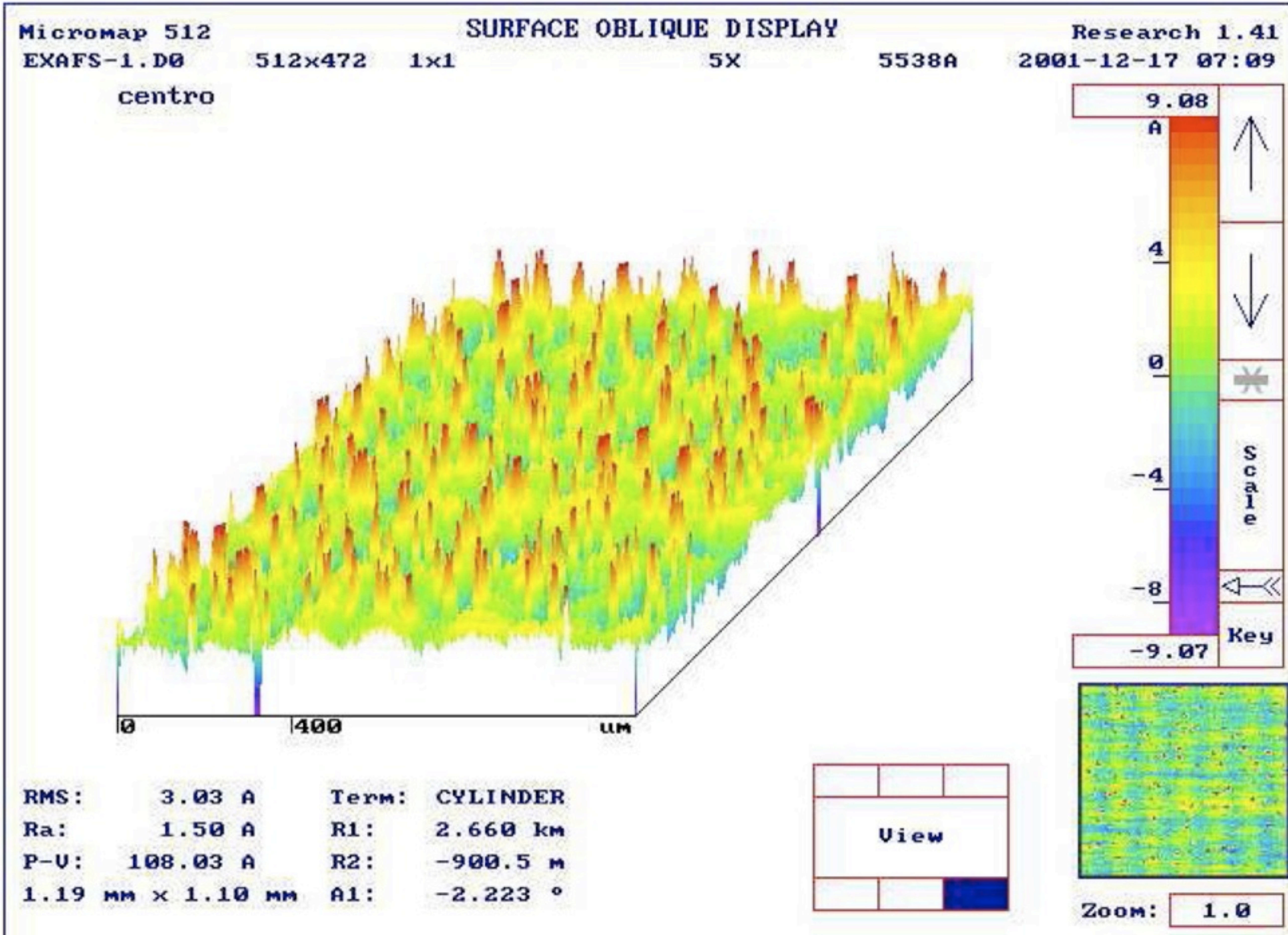


Mirror defects

Slope errors:
every deviation than from the ideal surface with period larger than ~ 1,2 mm
Typical definition is mrad or arcsec rms.
Alternative definition is λ/10 or λ/20 and so on… P-V or rms
used for normal incidence mirror or “poorer” quality mirrors

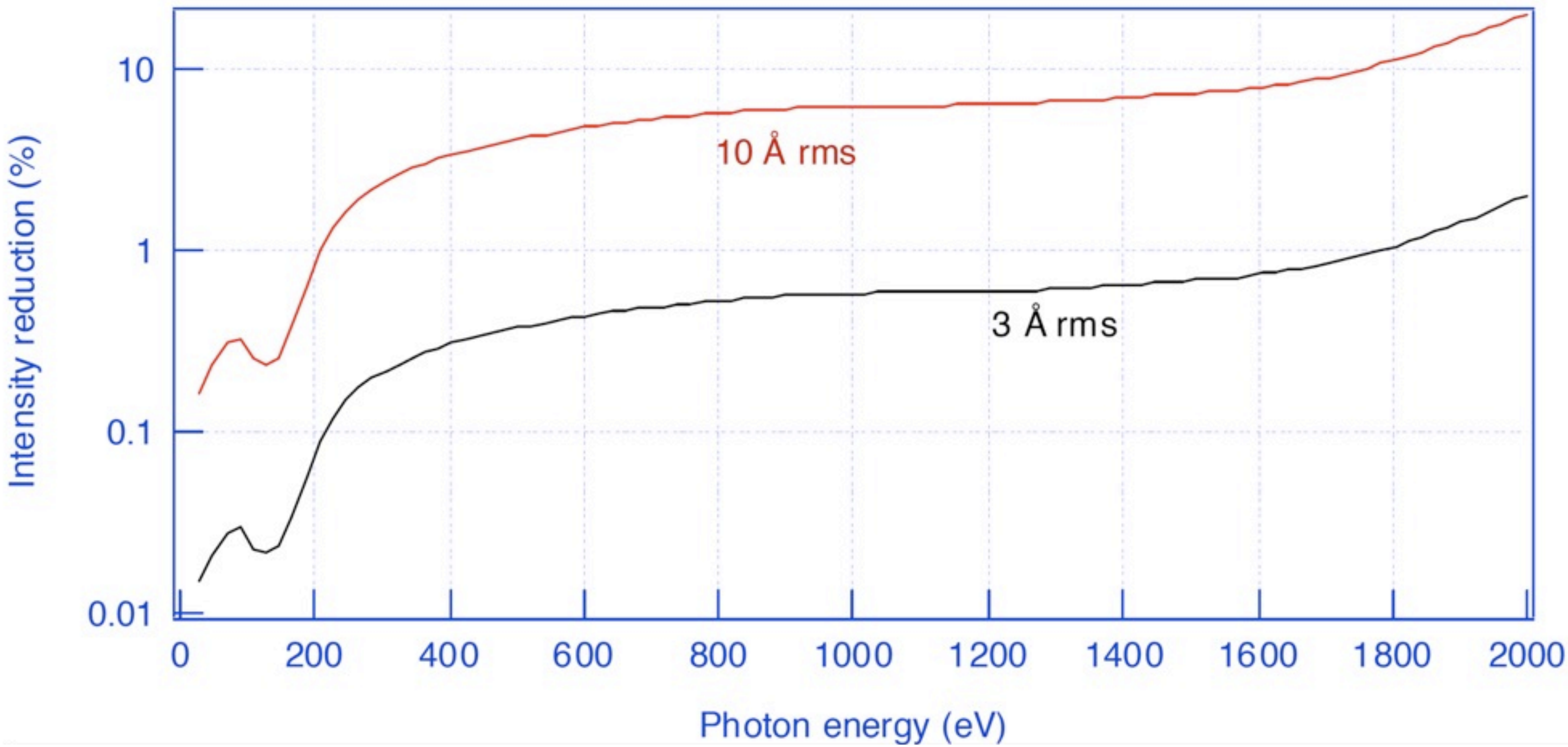
Roughness:
every deviation from the ideal surface with period smaller than ~ 0.5-1 mm
Typical definition is Å rms.
Alternative definition is surface quality 20-10 or 10-5 (scratch-dig)
used for normal incidence mirror or “poorer” quality mirrors
A dig is nearly equal in terms of its length and width. A scratch could be much longer than wide 20-10 means 20/1000 of mm max scratch width 10/100 mm max dig dimension

Roughness



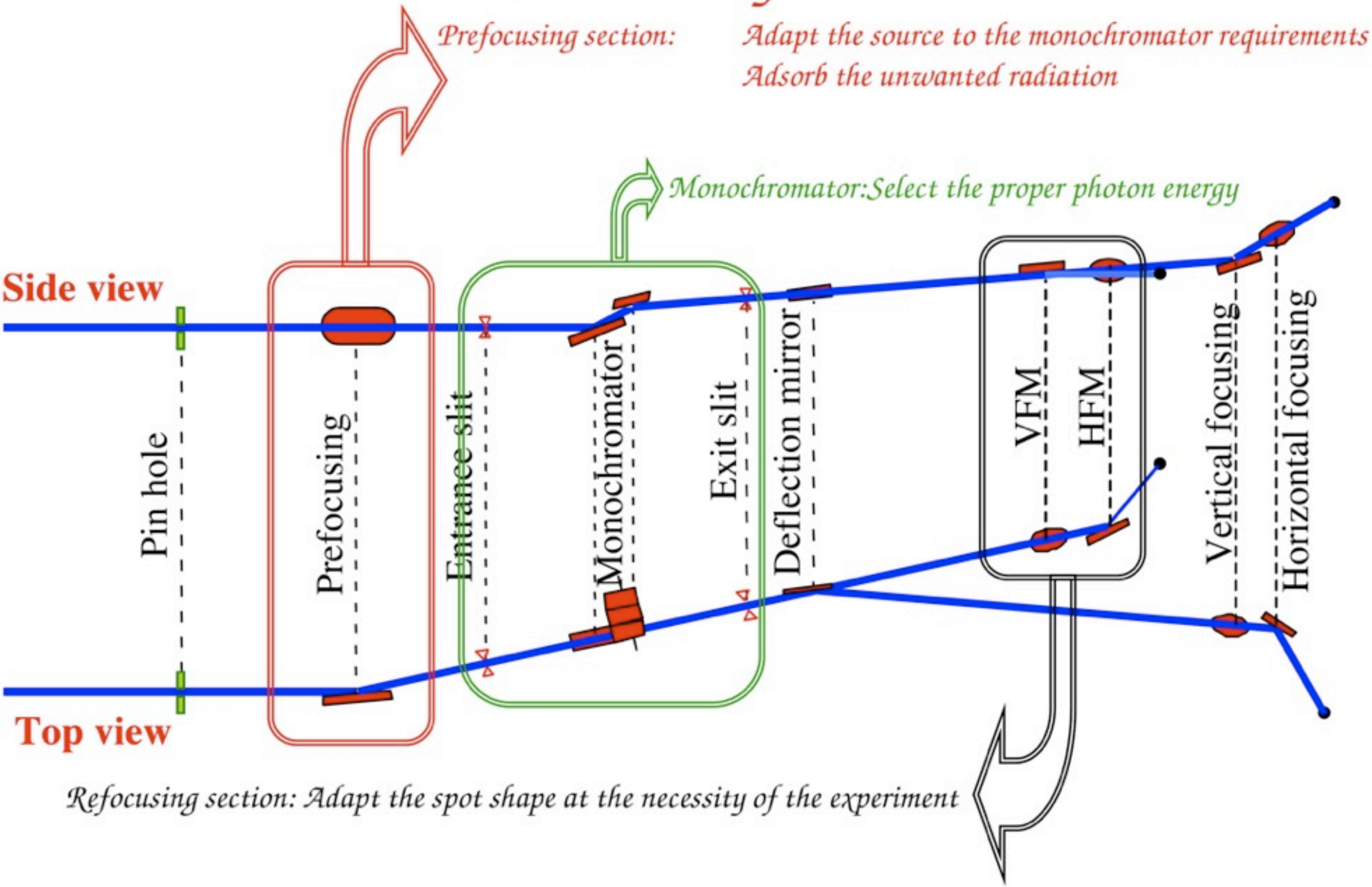
Micromap 512
SURFACE OBLIQUE DISPLAY
Research 1.41
EXAFS-1.D0
512x472
1x1
5X
5538A
2001-12-17 07:09
centro
9.08
A
4
0
-4
-8
-9.07
Scale
Key
0
400
um
RMS: 3.03 A
Ra: 1.50 A
P-V: 108.03 A
1.19 mm x 1.10 mm
Term: CYLINDER
R1: 2.660 km
R2: -900.5 m
A1: -2.223 °
View
Zoom: 1.0

Roughness

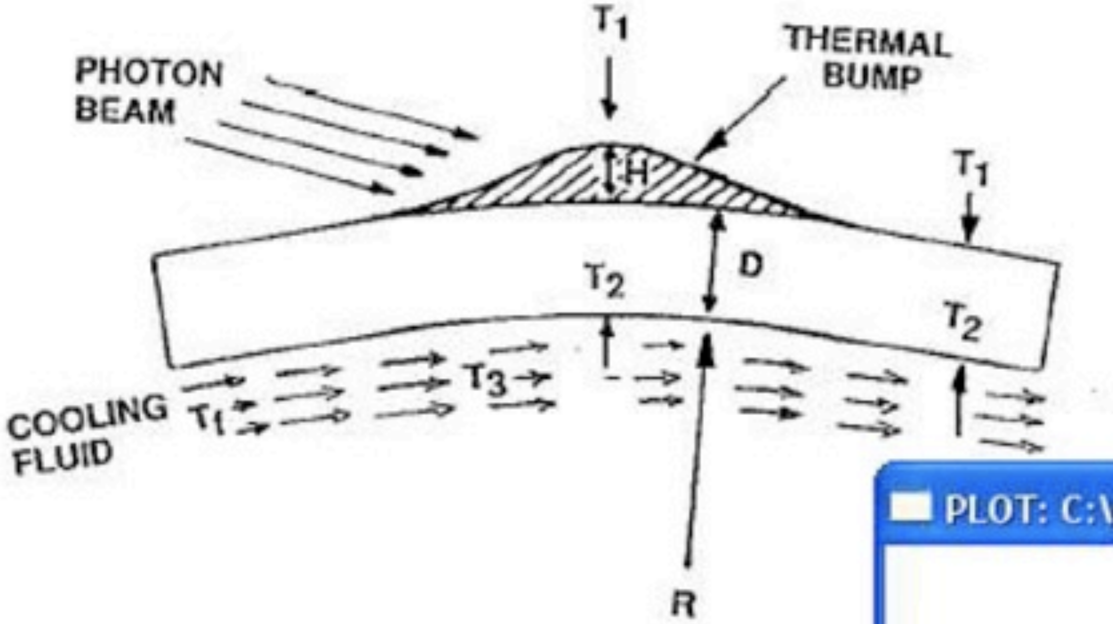


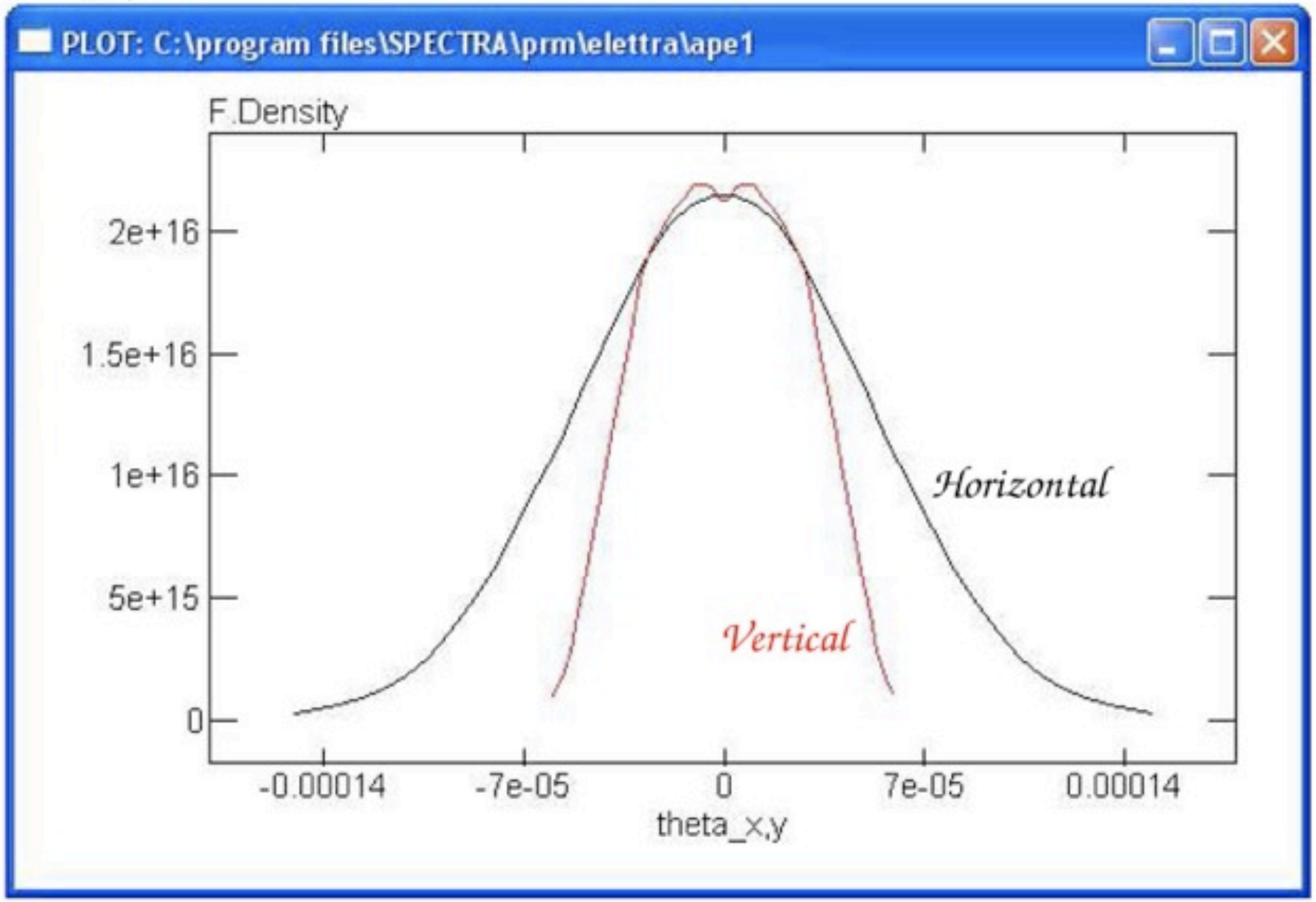
Shape	Spherical/Flat	Toroidal/aspherical
Roughness (Å)	3 typical (1 best)	5 typical (3 best)

Beamline layout



Thermal deformation





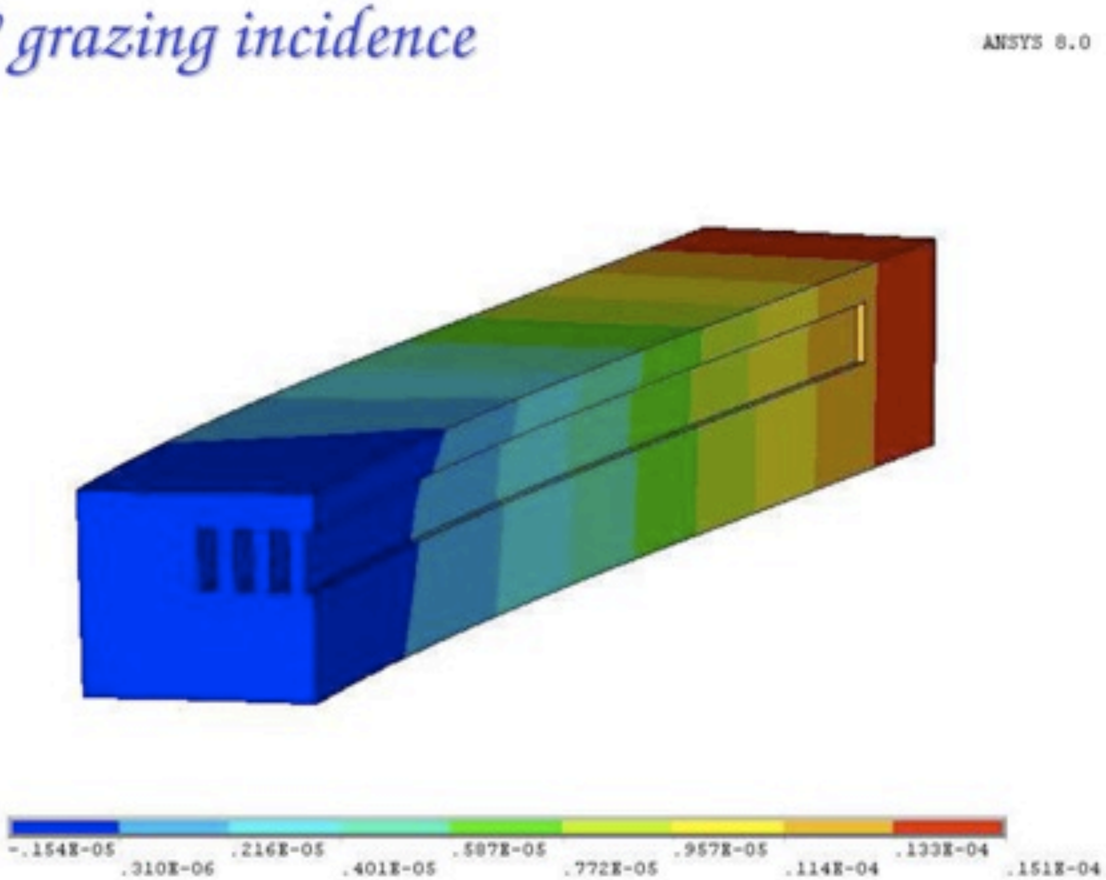
Mechanical and thermal properties of selected mirror materials

	Density (g/cm^3)	Young's modulus (GPa)	Thermal expansion α (ppm/˚C)	Thermal conductivity k (W/m/˚C)	Figure of merit k/α
Fused silica	2.19	73	0.50	1.4	2.80
Zerodur	2.53	92	0.05	1.60	32.00
Silicon	2.33	131	2.60	156	60.00
SiC CVD	3.21	461	2.40	198	82.50
Aluminum	2.70	68	22.5	167	7.42
Copper	8.94	117	16.5	391	23.70
Glidcop	8.84	130	16.6	365	21.99
Molybdenum	10.22	324.8	4.80	142	29.58

Induced slope errors for a 400W source

glidcop

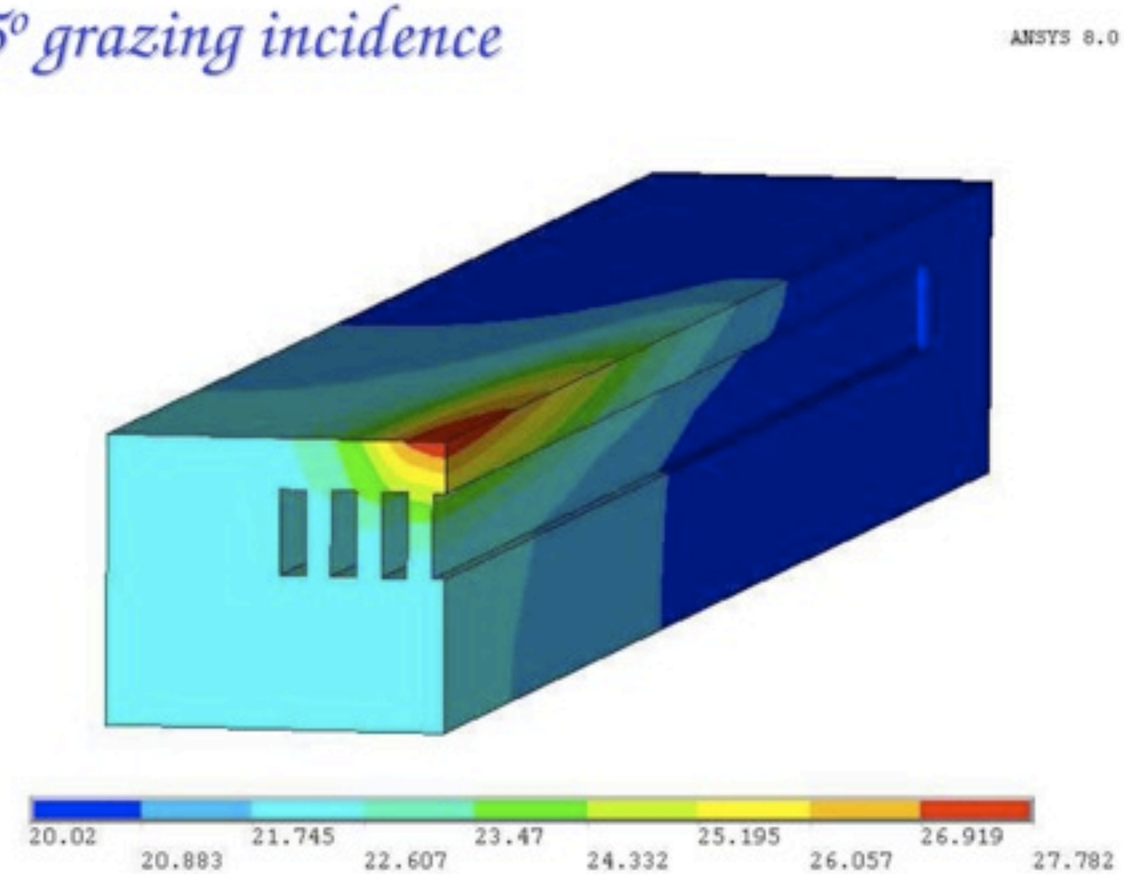
1.5° grazing incidence



Δh=17μm slope 26 μrad

3GeV Synchrotron source
6.6 cm period undulator K_{max}=5.7
BL6.1

1.5° grazing incidence



ΔT=7.7°

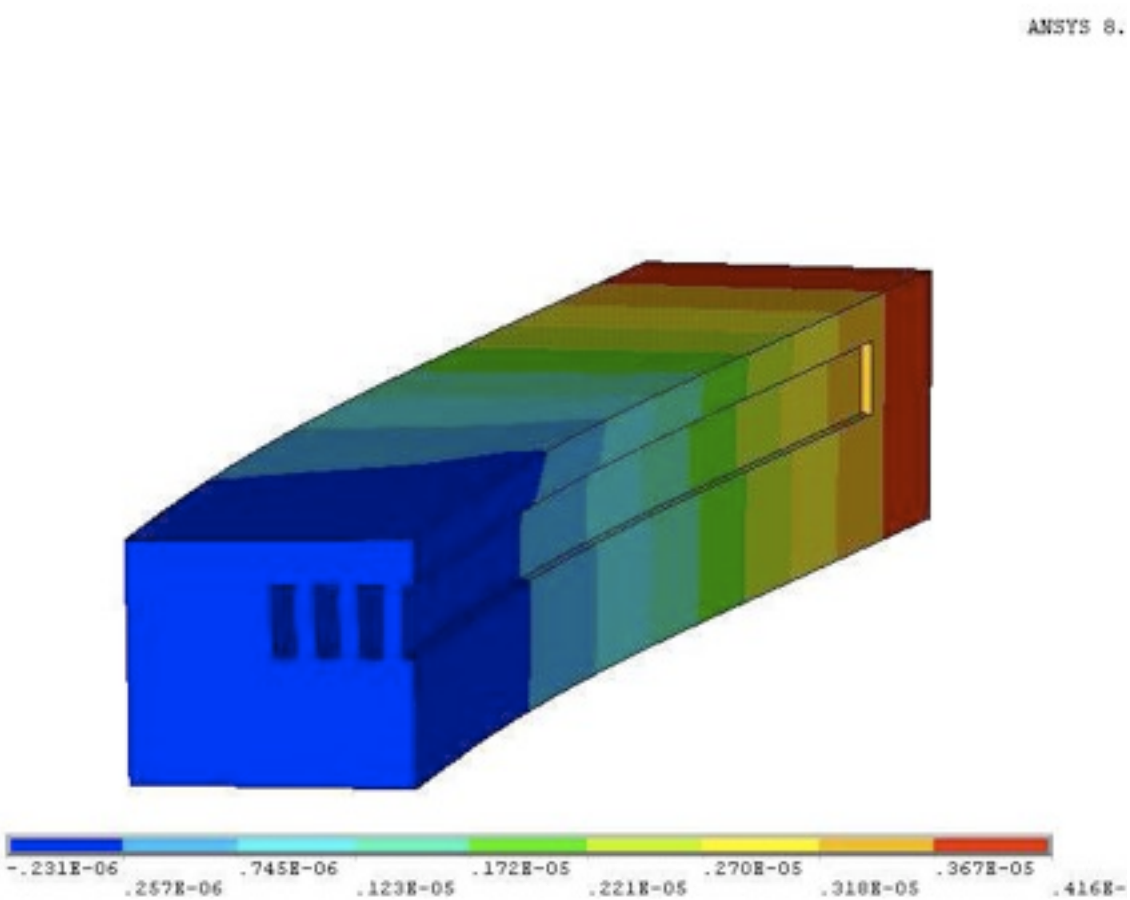
Induced slope errors for a 400W source

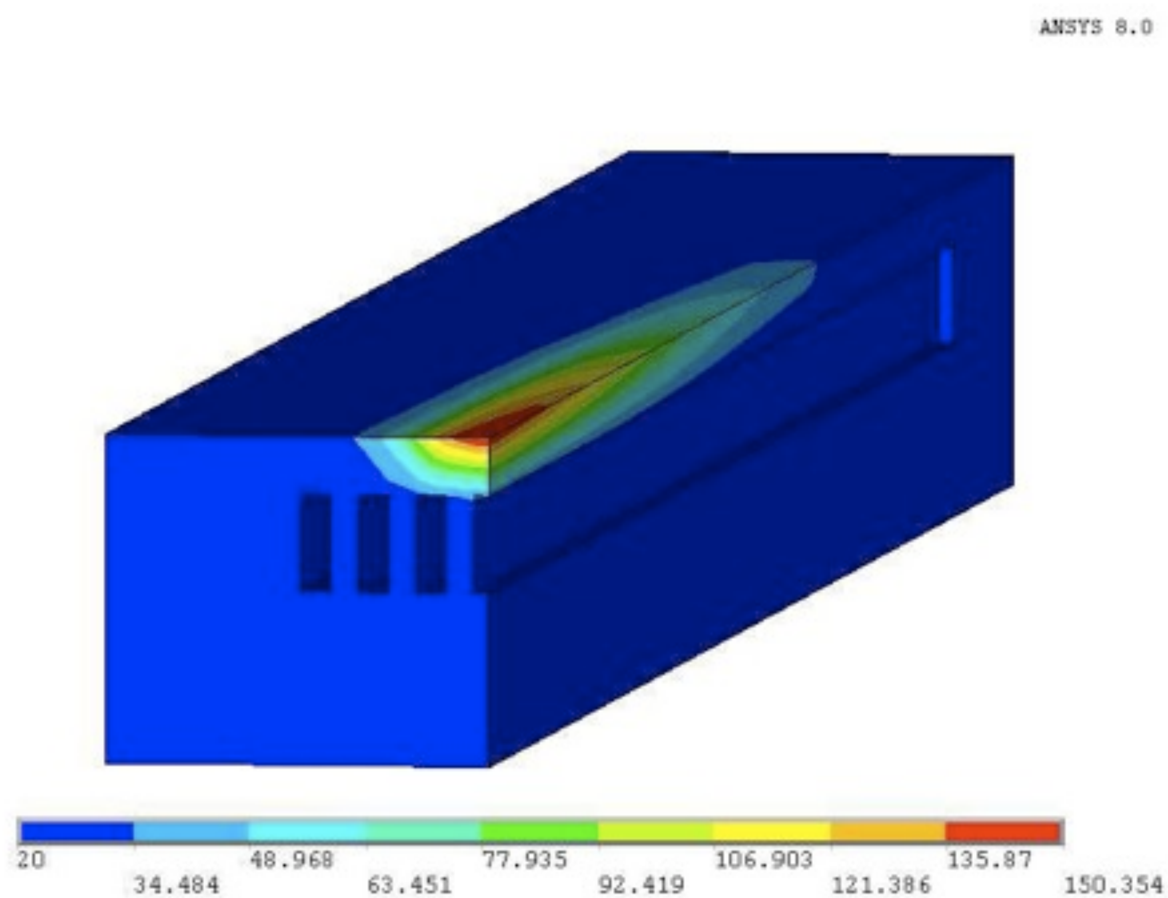
	Density (g/cm^3)	**Young's modulus** (GPa)	**Thermal expansion** α (ppm/˚C)	**Thermal conductivity** k (W/m/˚C)	**Figure of merit** k/α
Glidcop	8.84	130	16.6	365	21.99
Molybdenum	10.22	324.8	4.80	142	29.58
SuperInvar	8.13	145	0.06	10.5	175.00

Induced slope errors for a 400W source

	Density (g/cm³)	**Young's modulus** (GPa)	**Thermal expansion** α (ppm/˚C)	**Thermal conductivity** k (W/m/˚C)	**Figure of merit** k/α
Glidcop	8.84	130	16.6	365	21.99
Molybdenum	10.22	324.8	4.80	142	29.58
SuperInvar	8.13	145	0.06	10.5	175.00

SuperInvar

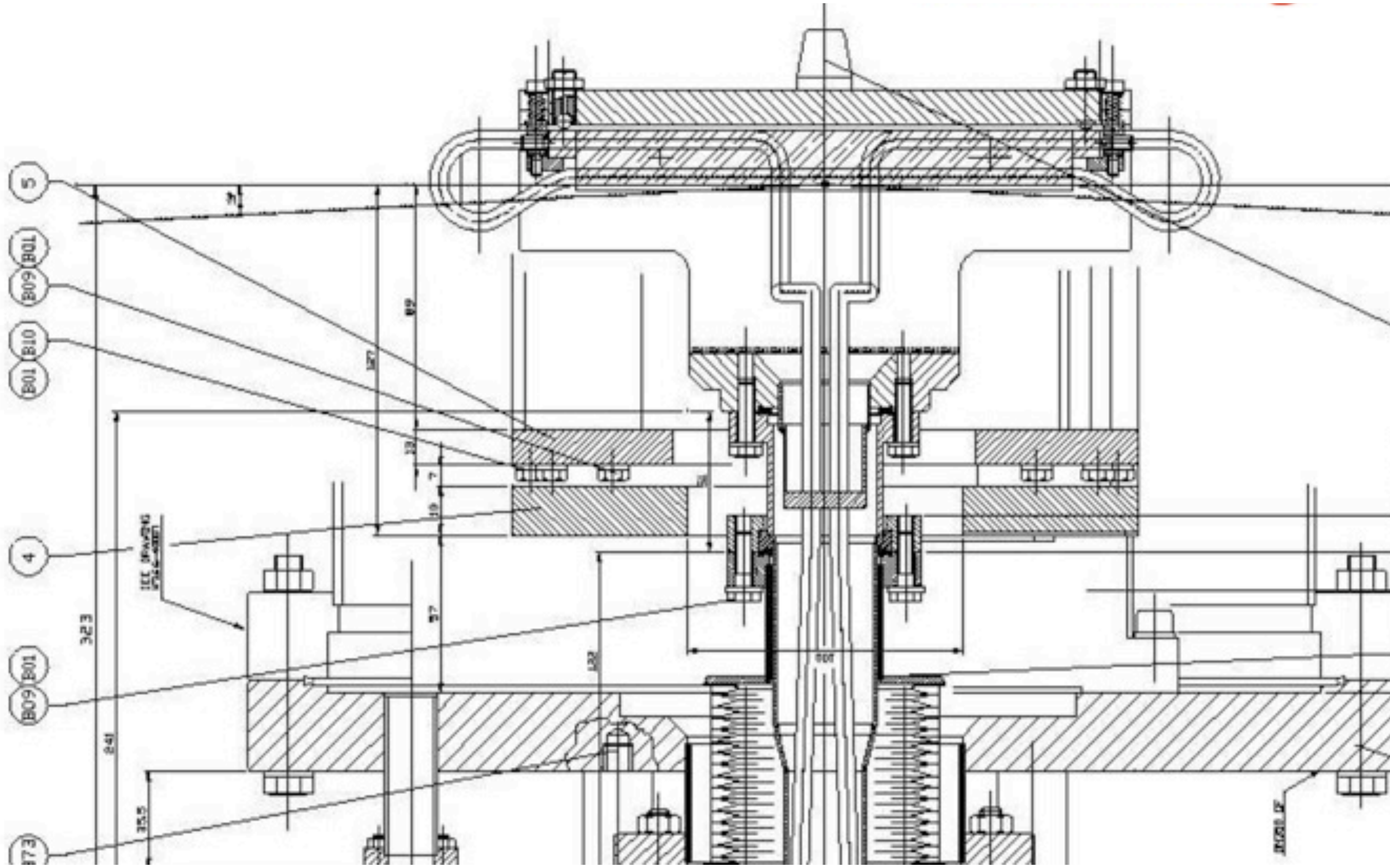


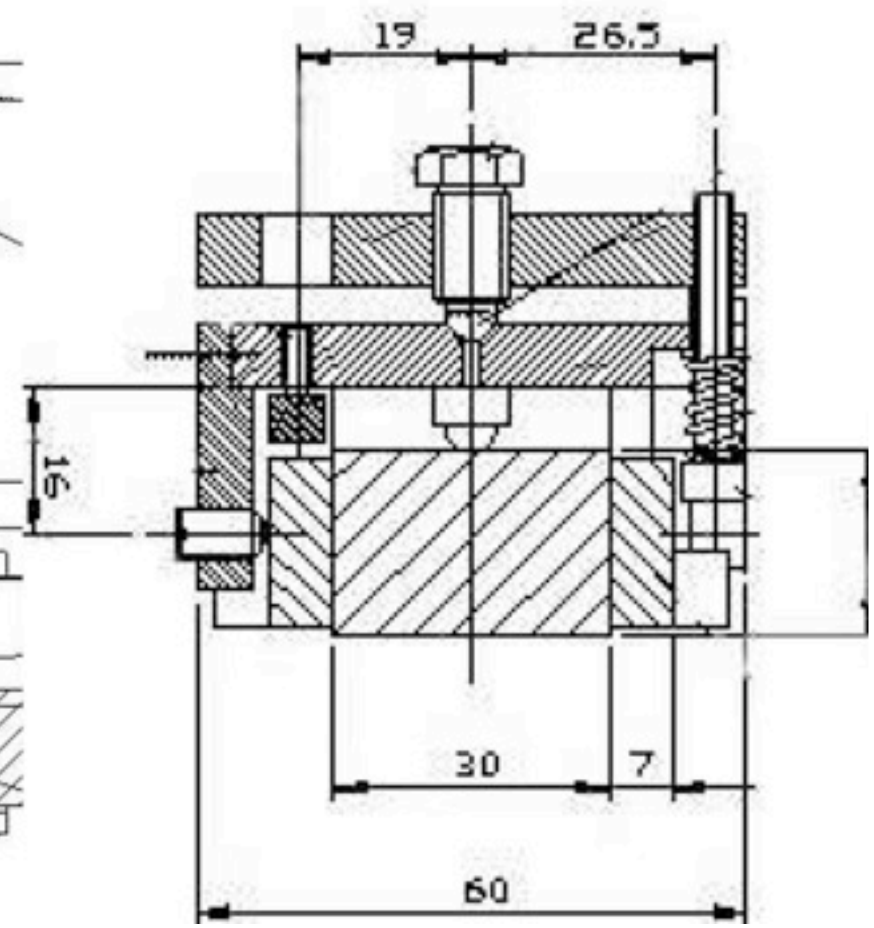


$\Delta h = 6 \mu m$

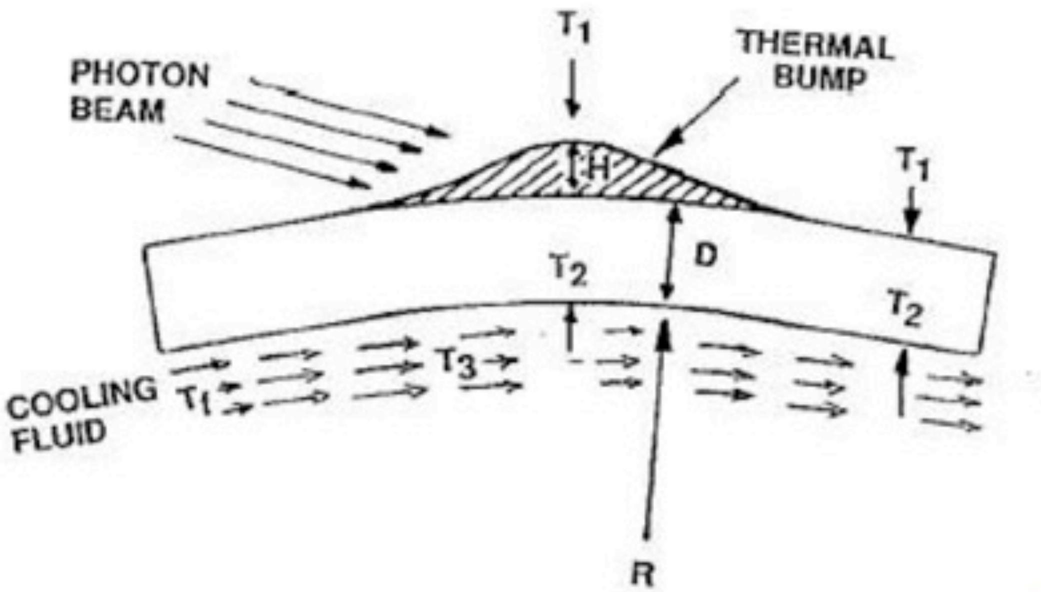
$\Delta T = 130^{\circ}C$

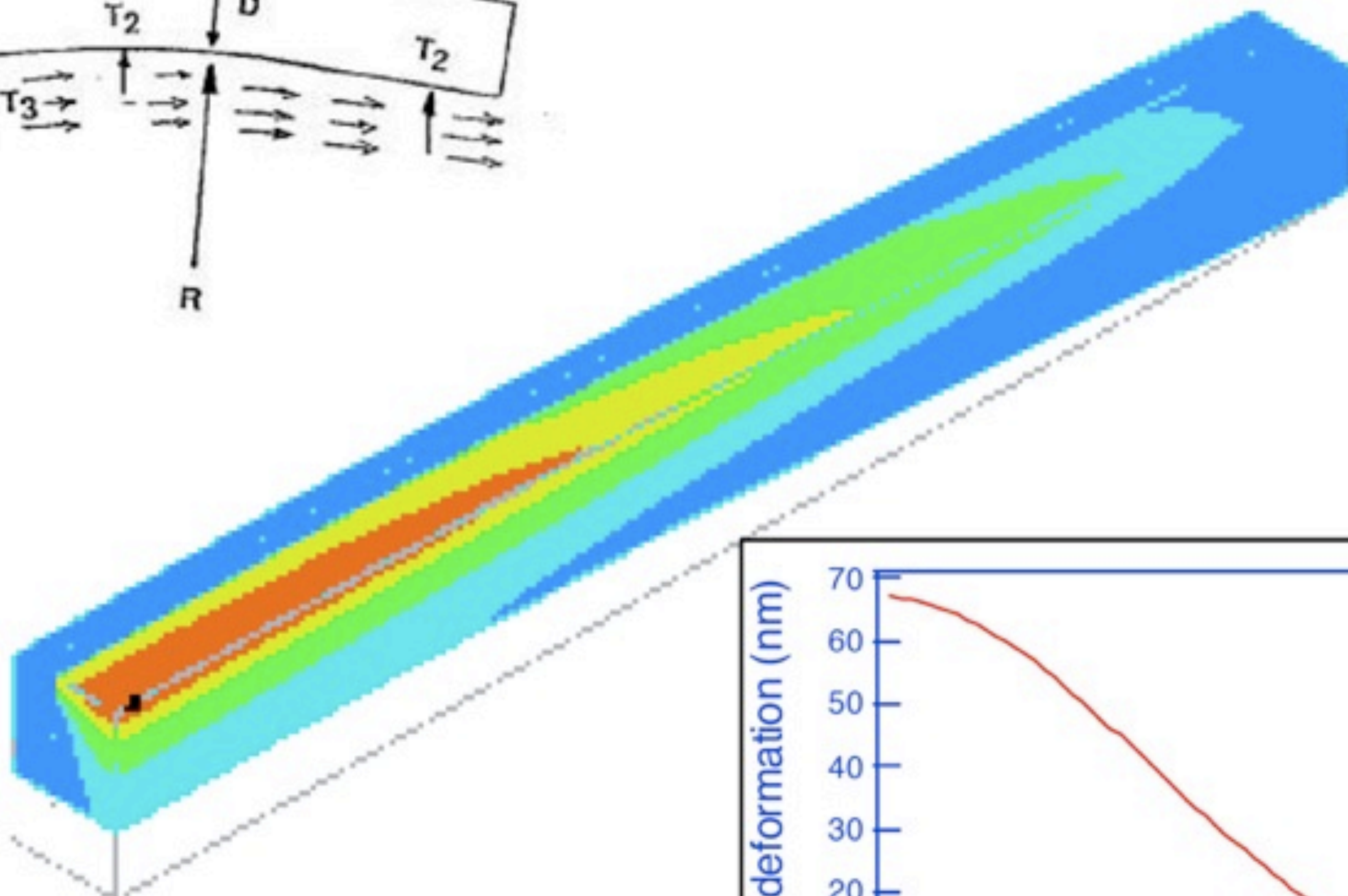
Side cooling

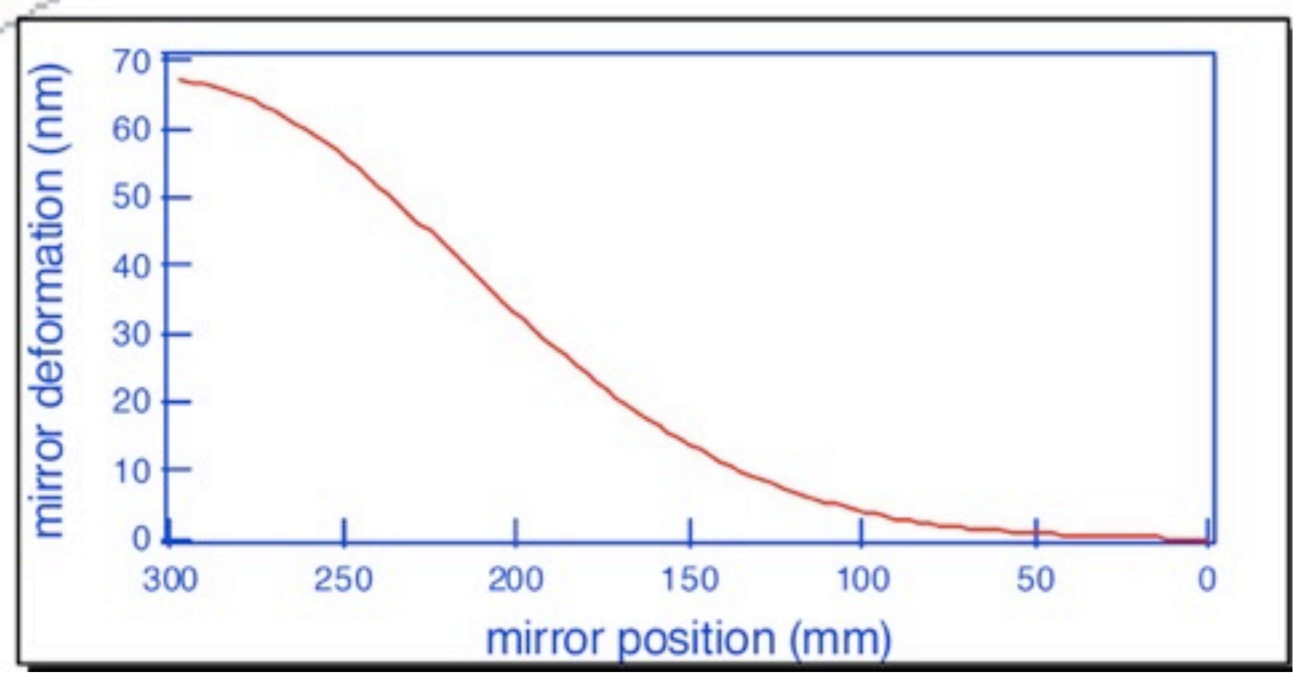




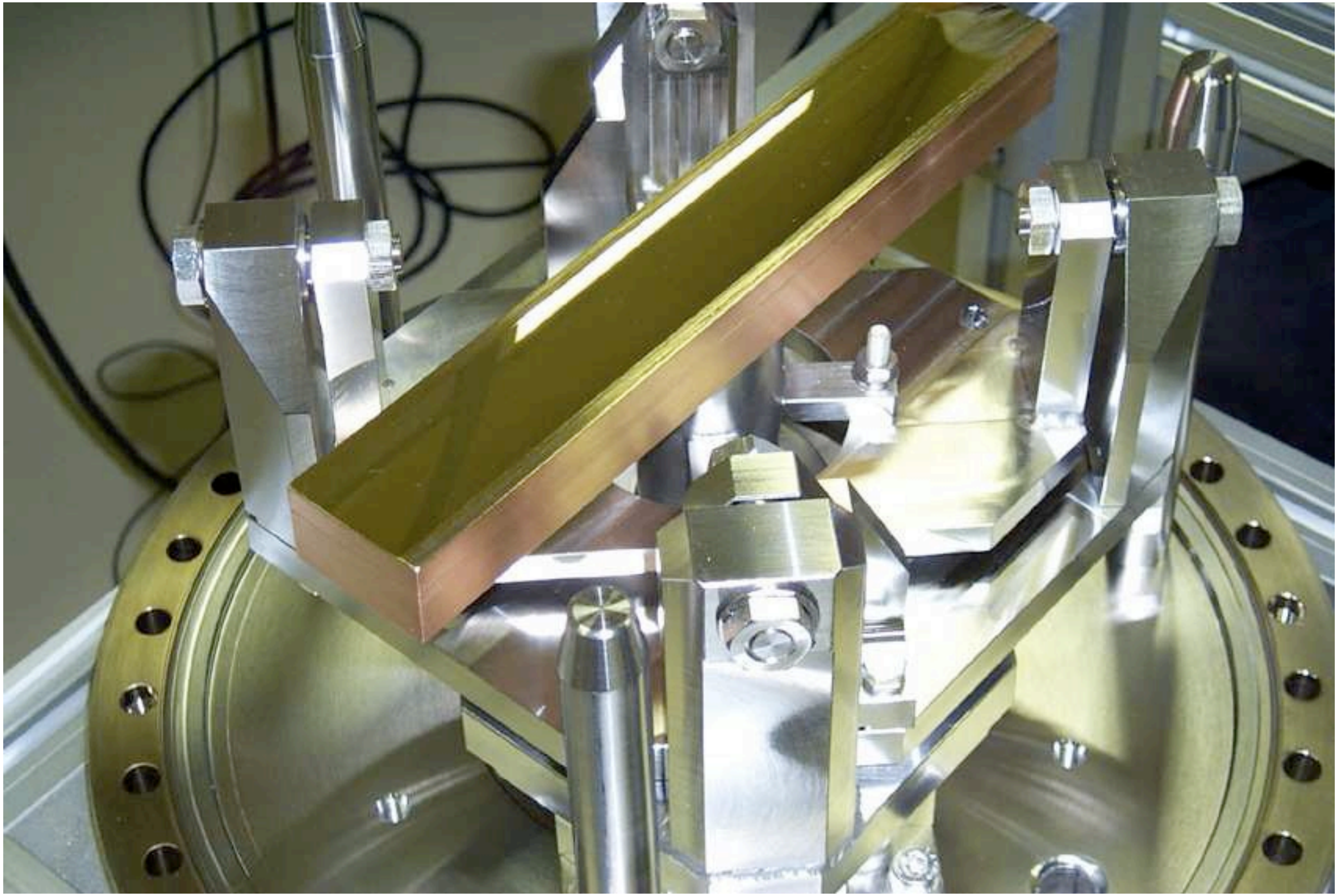
Side cooling



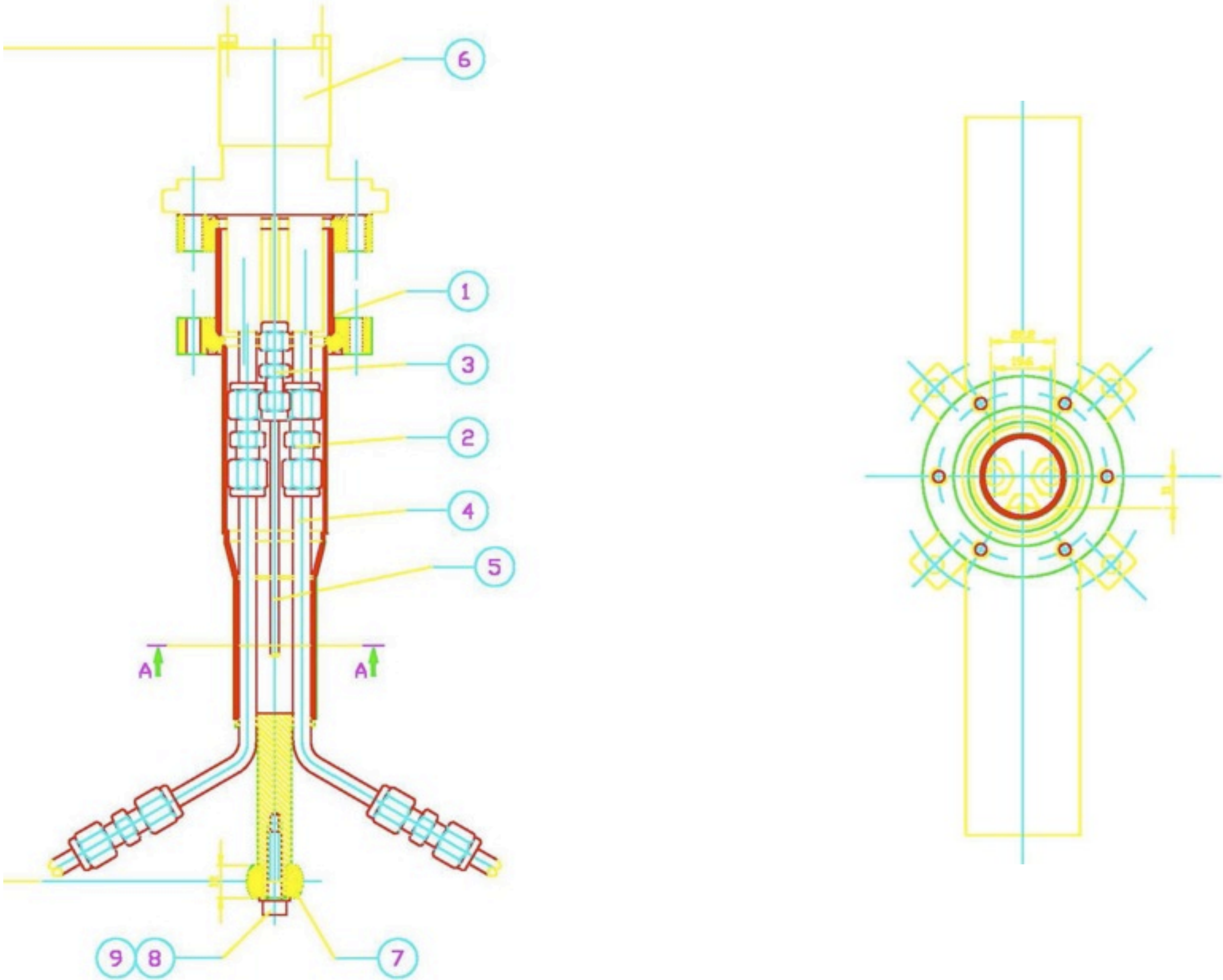




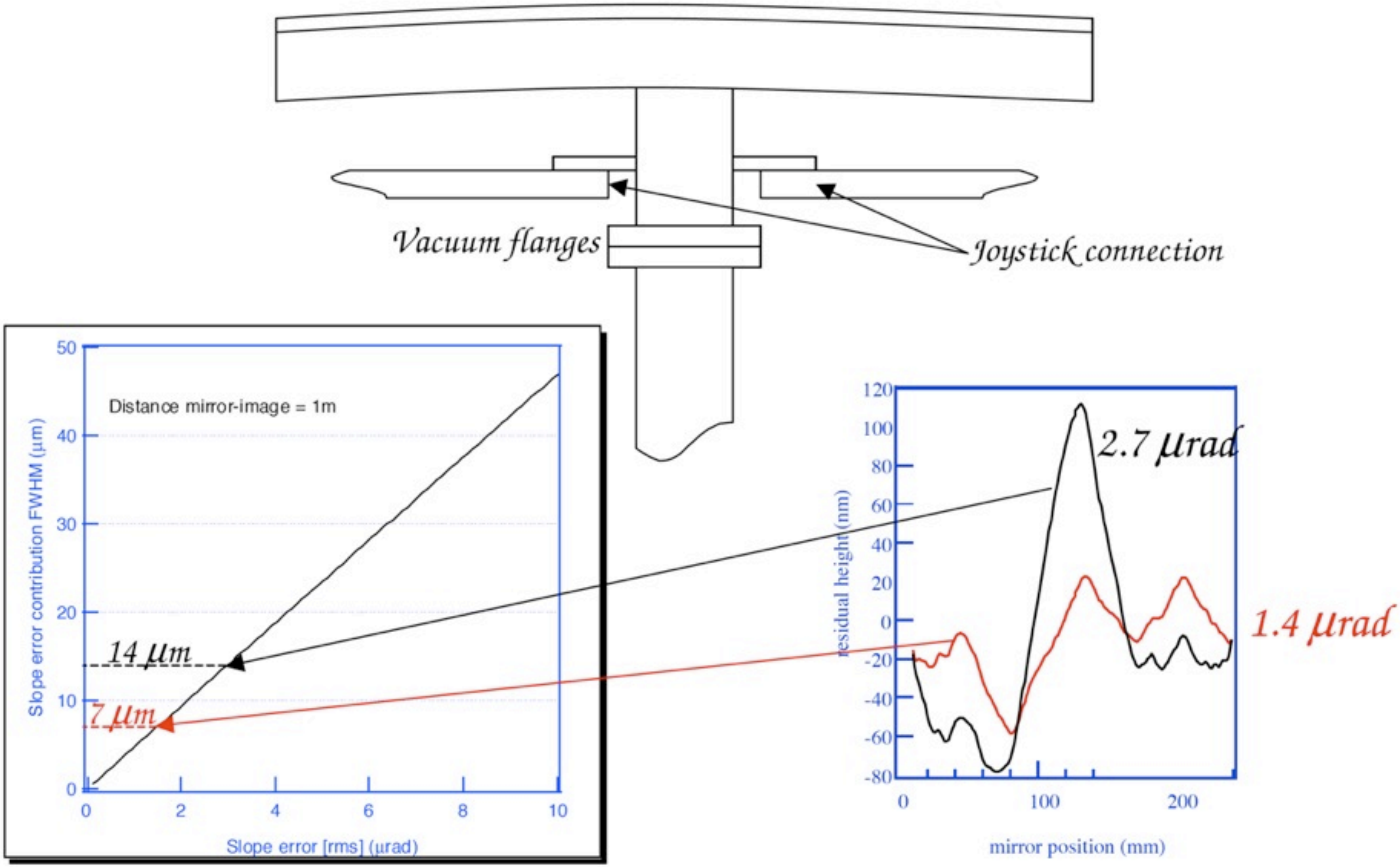
Internal cooling



Internal cooling



Internal cooling



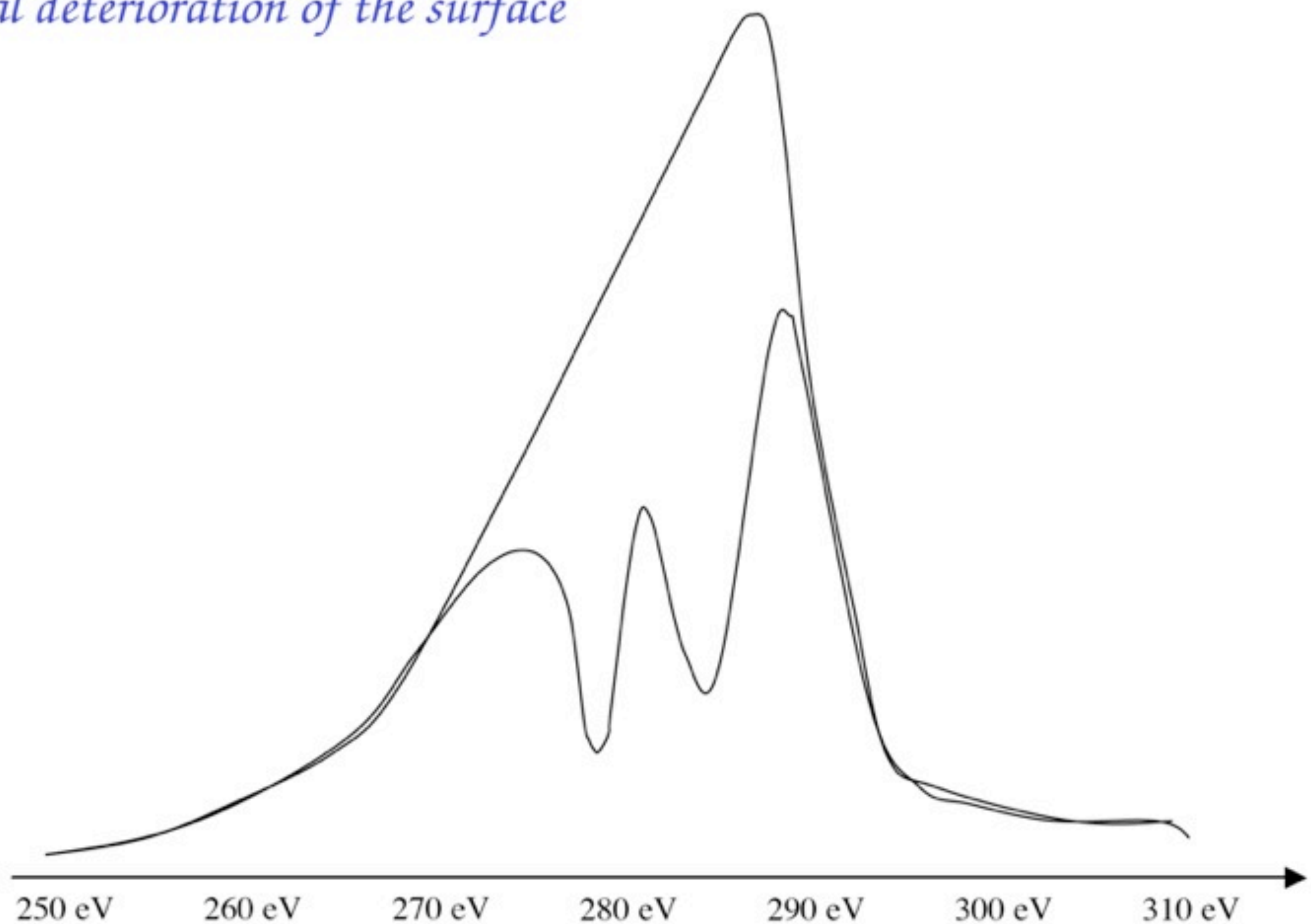
Carbon contamination

Effect of the contamination:

Strong adsorption at the carbon edge (≈270 eV)

Reduction of reflectivity due to enanchment of the surface roughness

general deterioration of the surface



Carbon contamination and cleaning

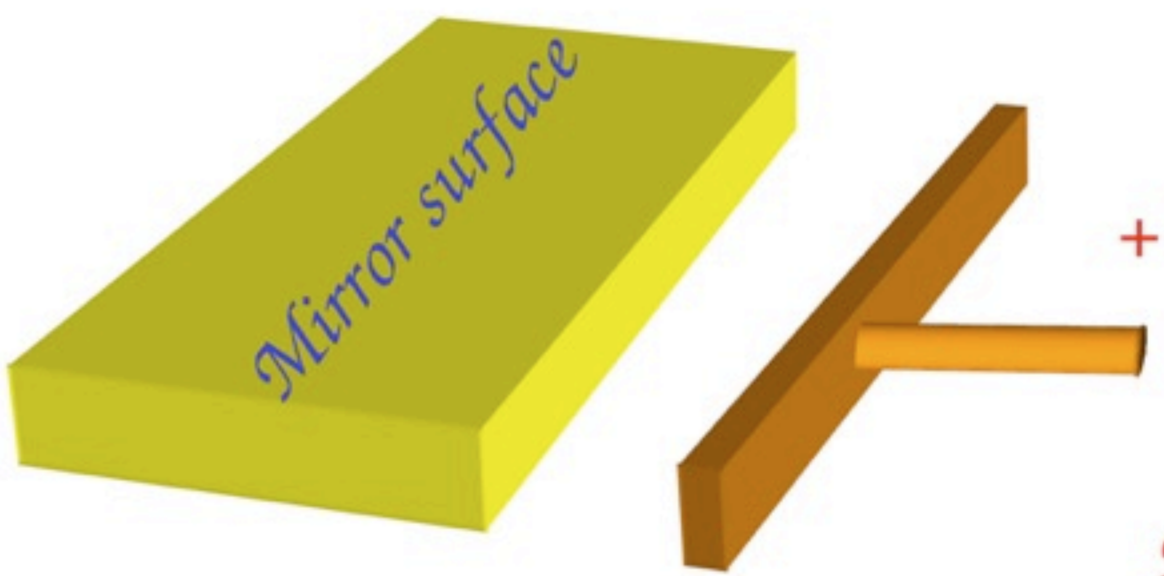
Contamination process:

- *Hydrocarbons adsorbed by the surface*
- *Cracking induced by the incoming radiation*
- *Formation of graphitic carbon layer (mixed C coumpond)*

Effect of the contamination:

- *Strong adsorption at the carbon edge (≈270 eV)*
- *Reduction of reflectivity due to enanchment of the surface roughness*
- *general deterioration of the surface*

UV lamp discharge

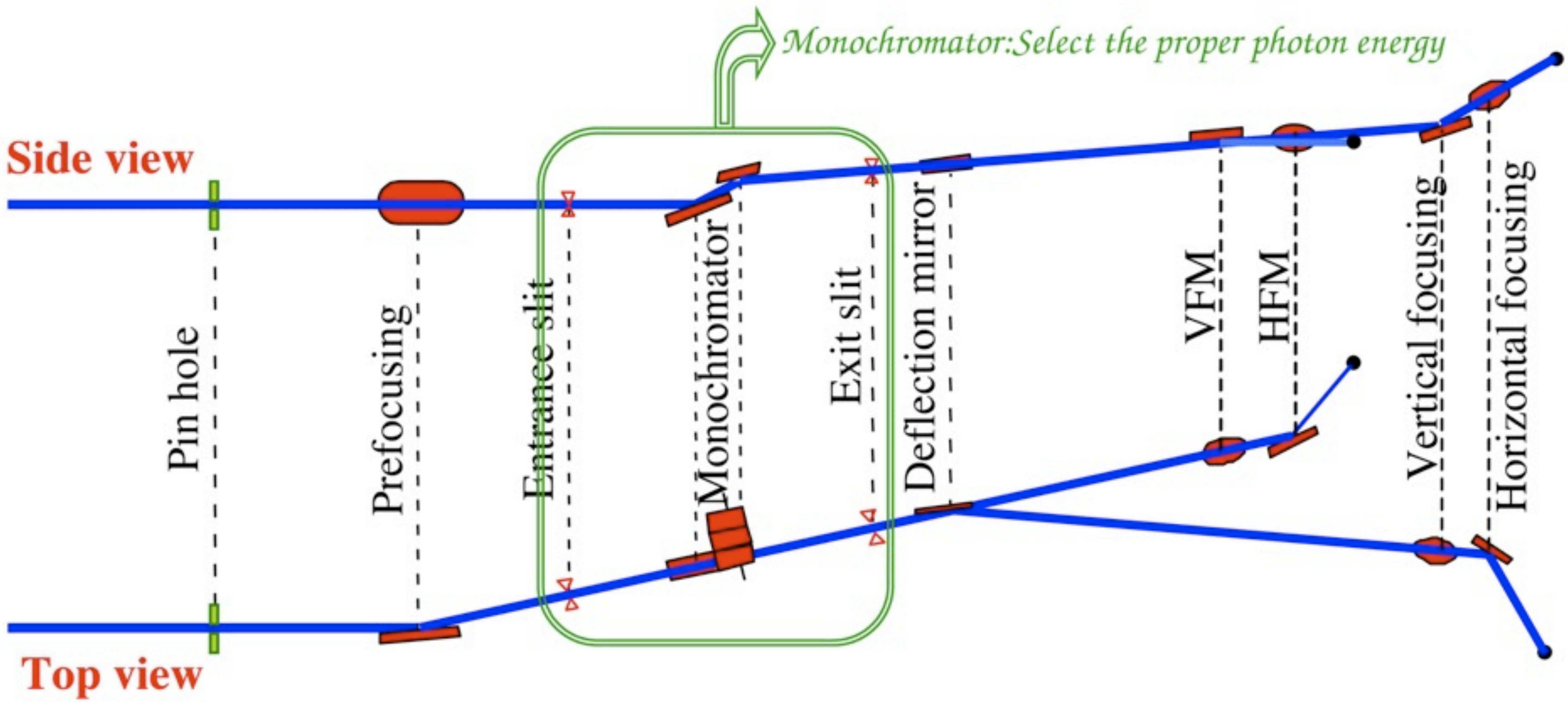


+ 300-500 V (DC)

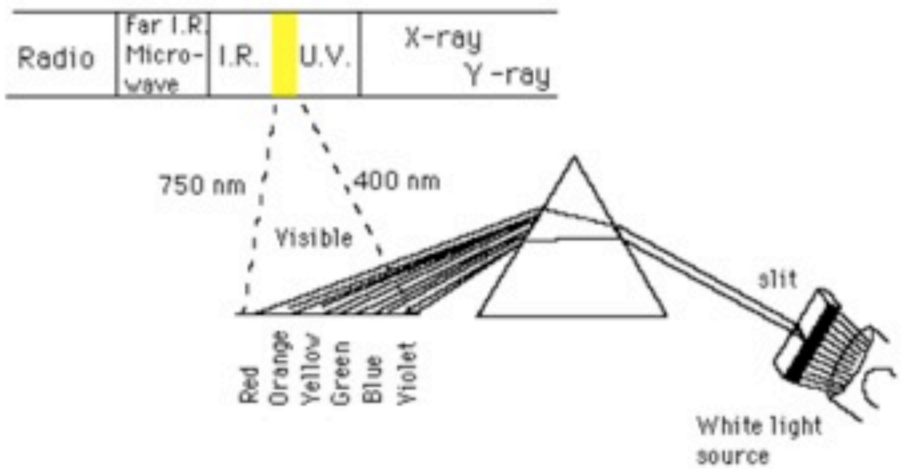
I 100 mA-1A

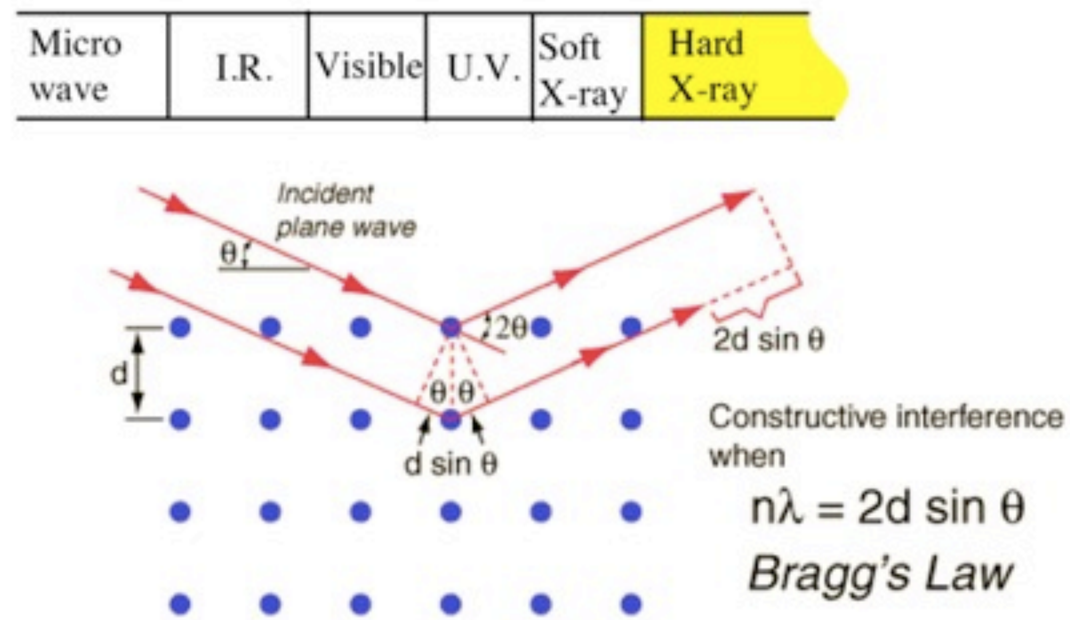
P 0.5-1 mbar O_2

Soft X-ray monochromators

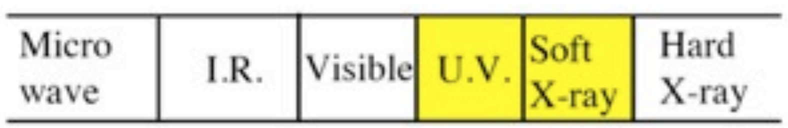


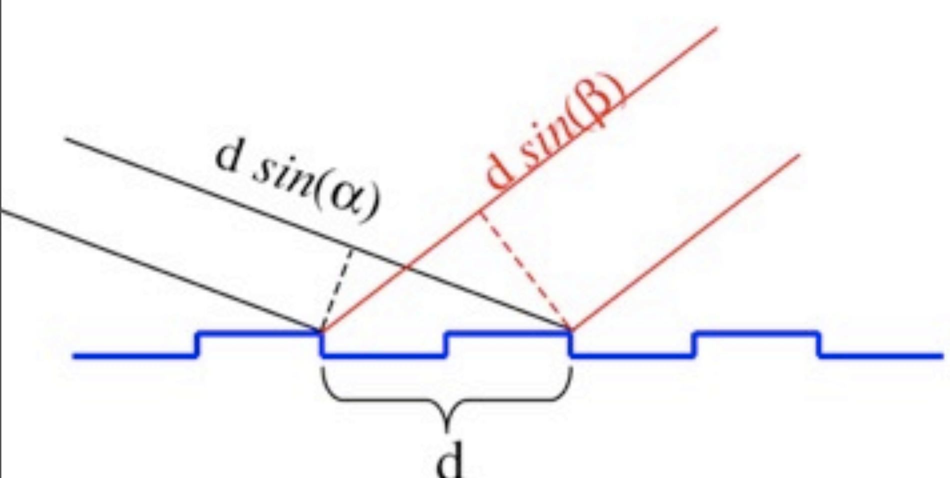
Soft X-ray monochromators

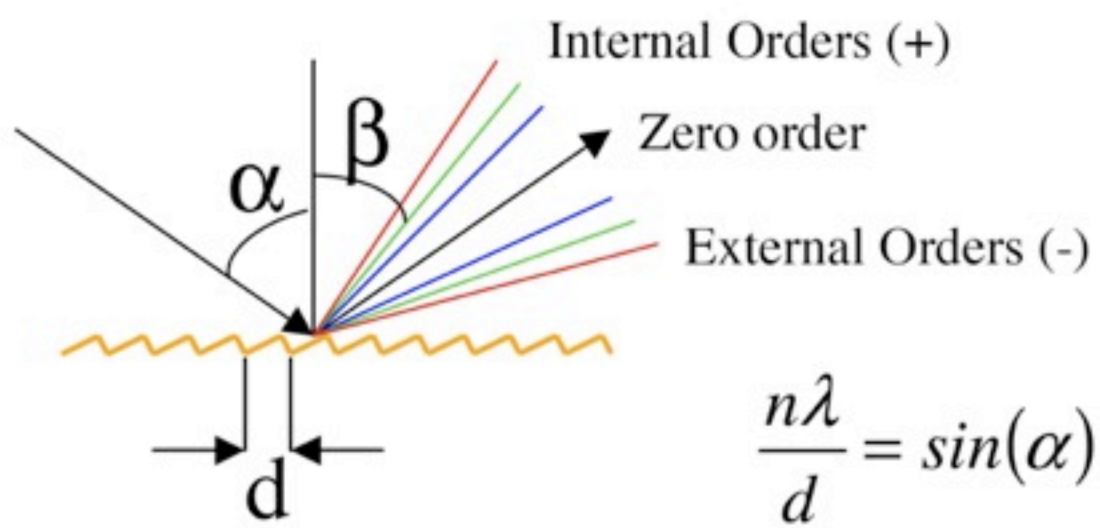




limit ~ 1-2 keV (1 nm)

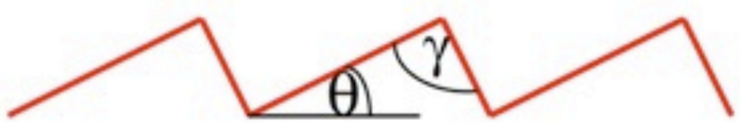




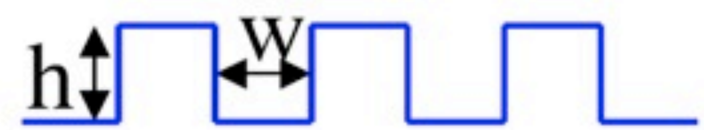


$$\frac{n\lambda}{d} = sin(\alpha) - sin(\beta)$$

Grating profiles

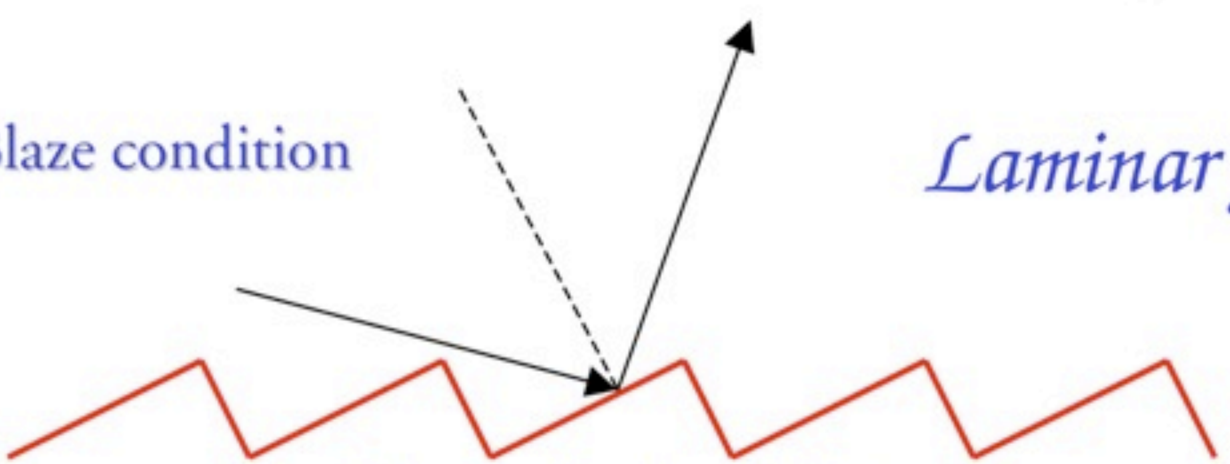


Blaze profile



Laminar profile

Blaze condition

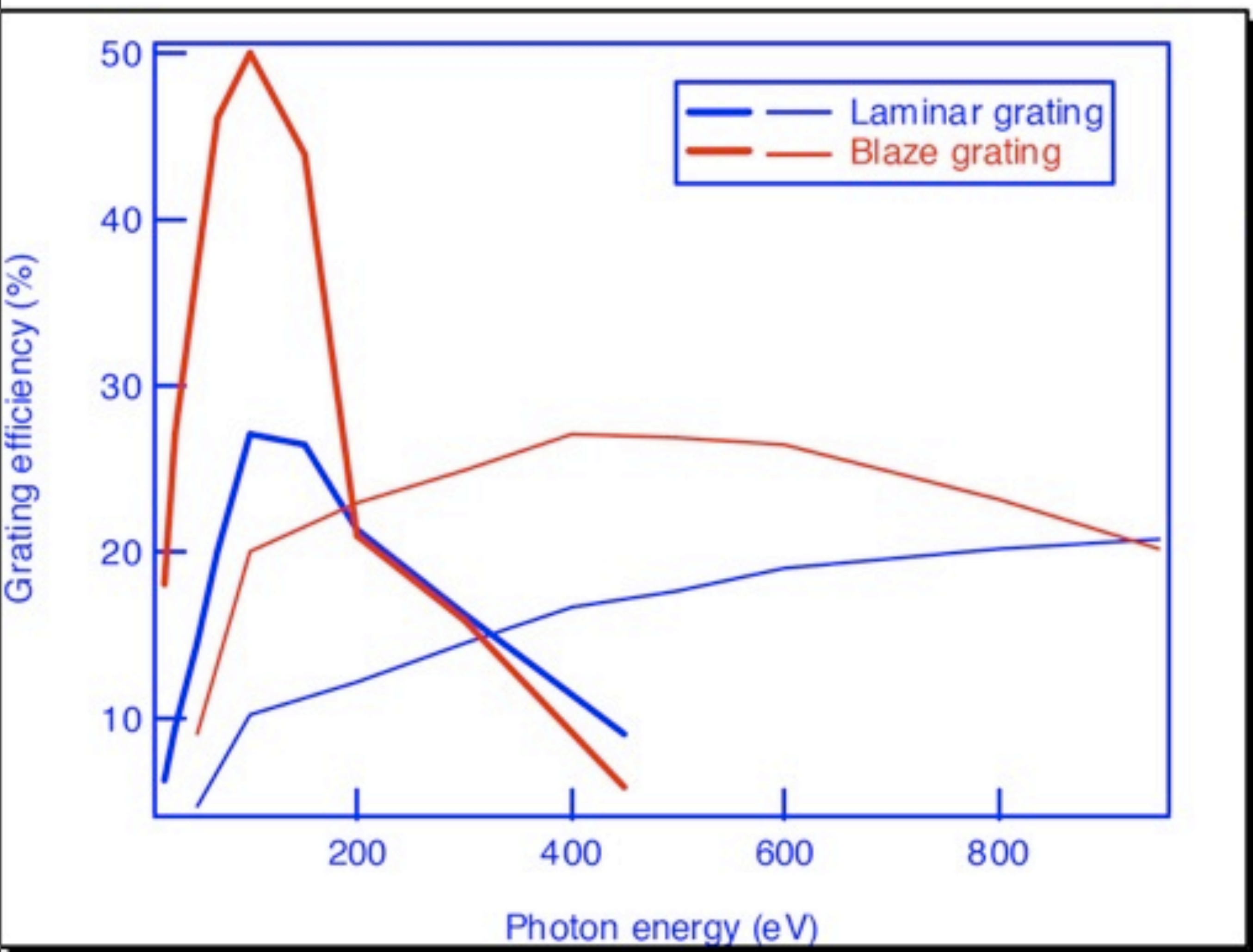


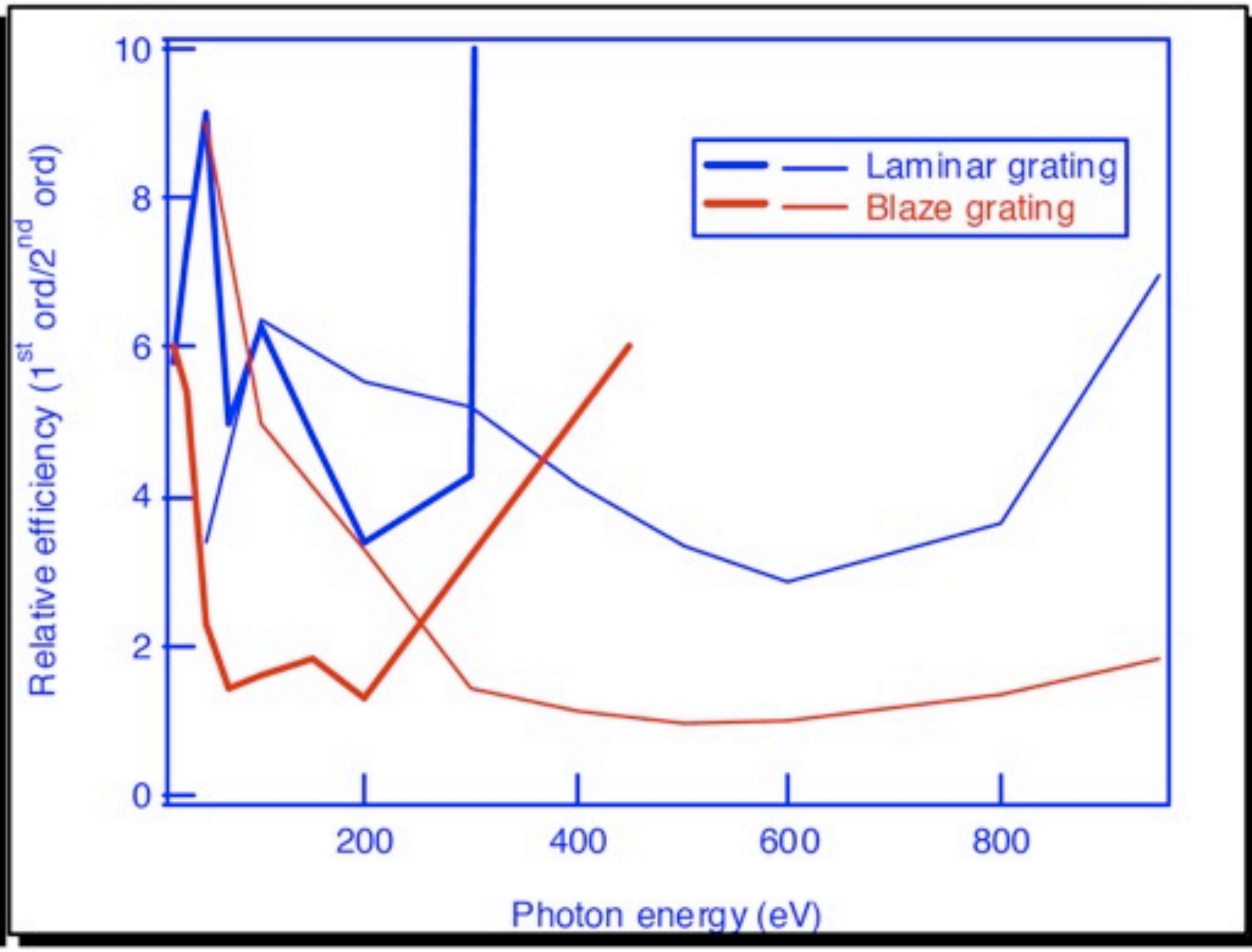
Blaze angle=(α+β)/2

Blaze gratings: *higher efficiency*

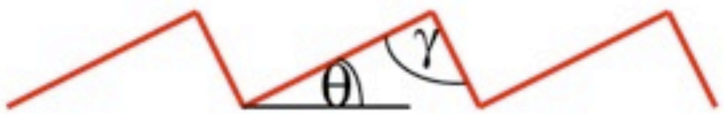
Laminar gratings: *Higher spectral purity*
Higher resolution

Grating profiles

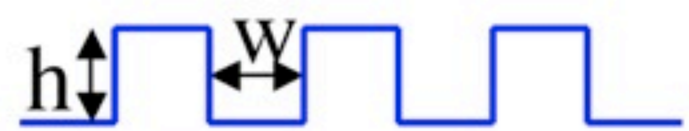




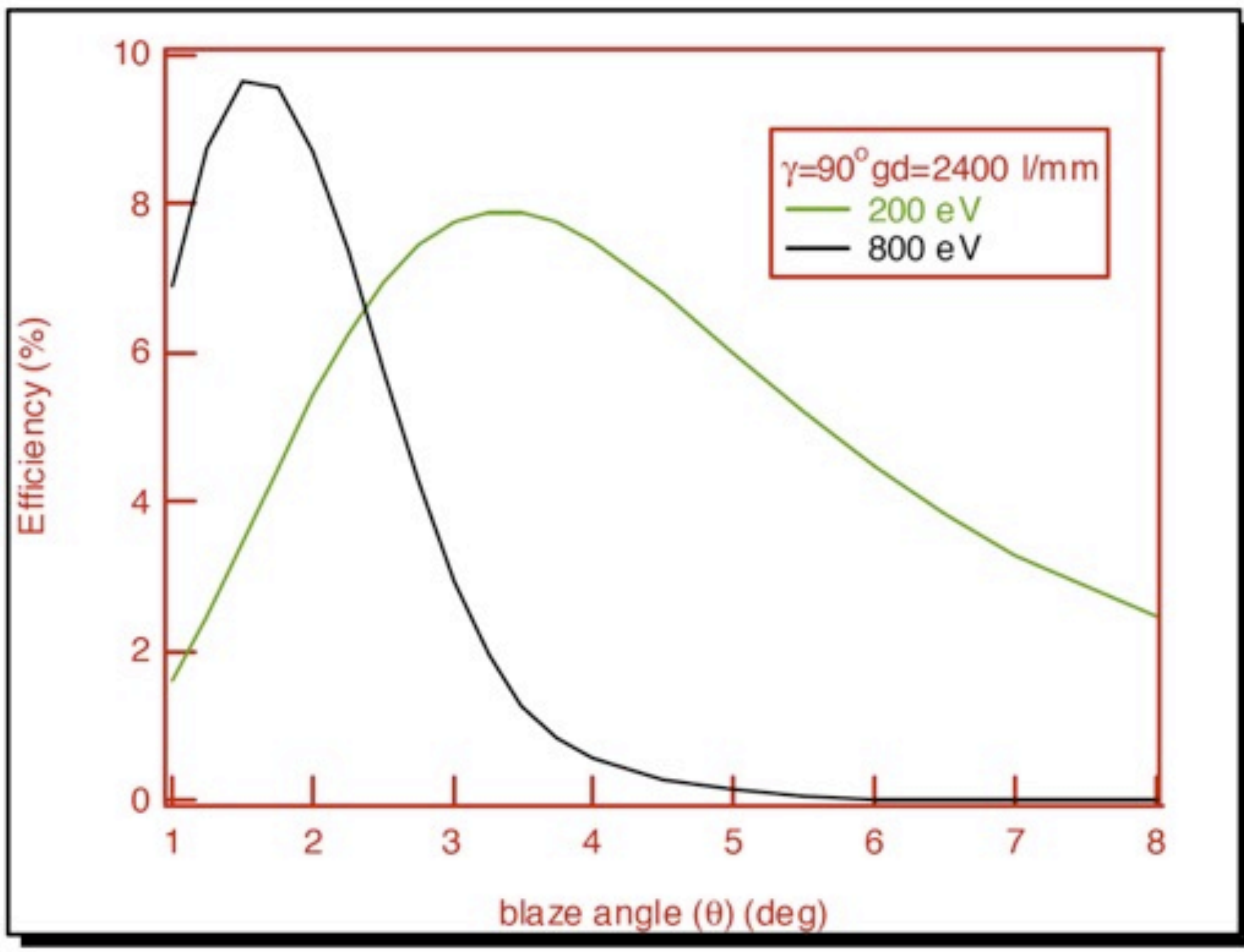
Grating profiles

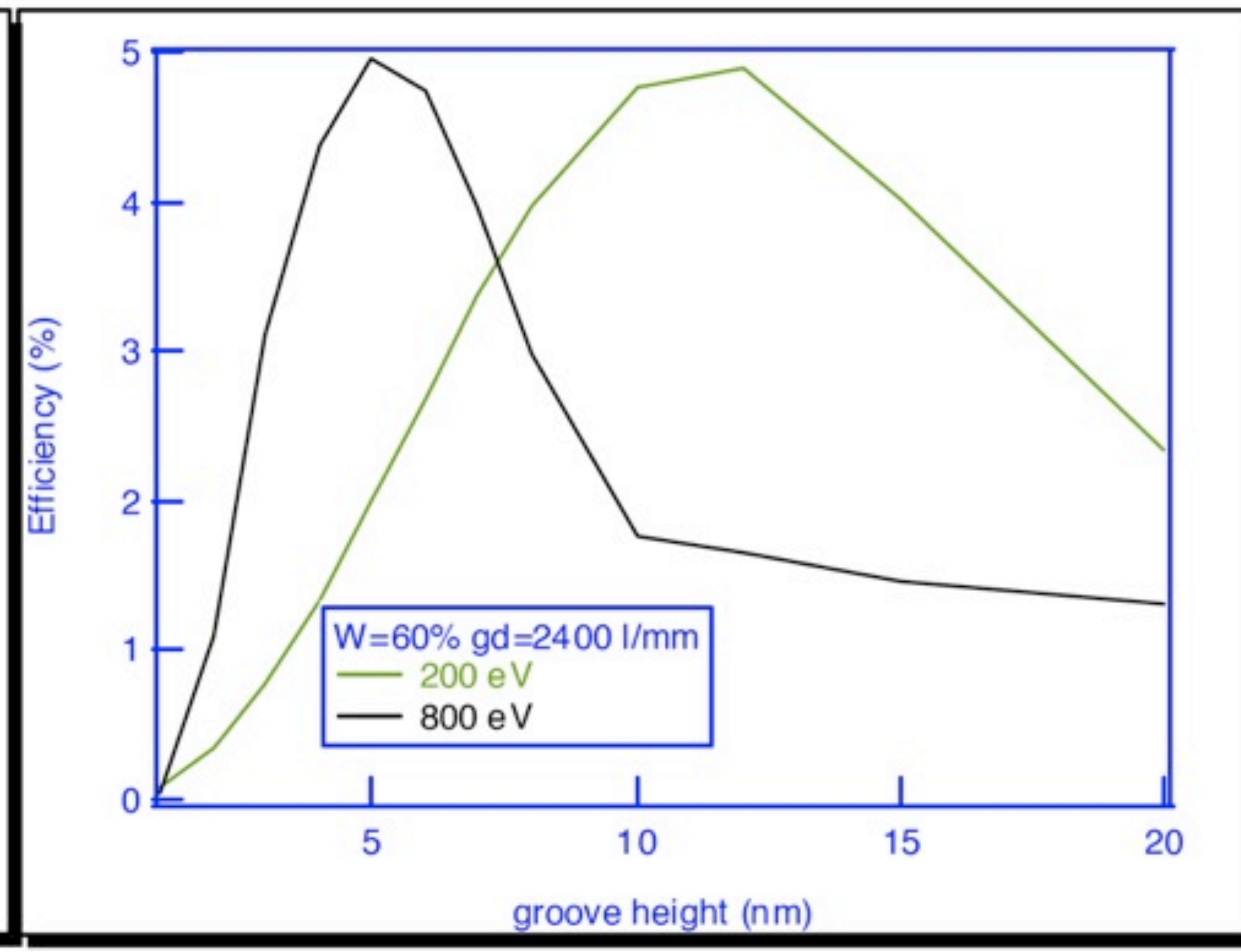


Blaze profile



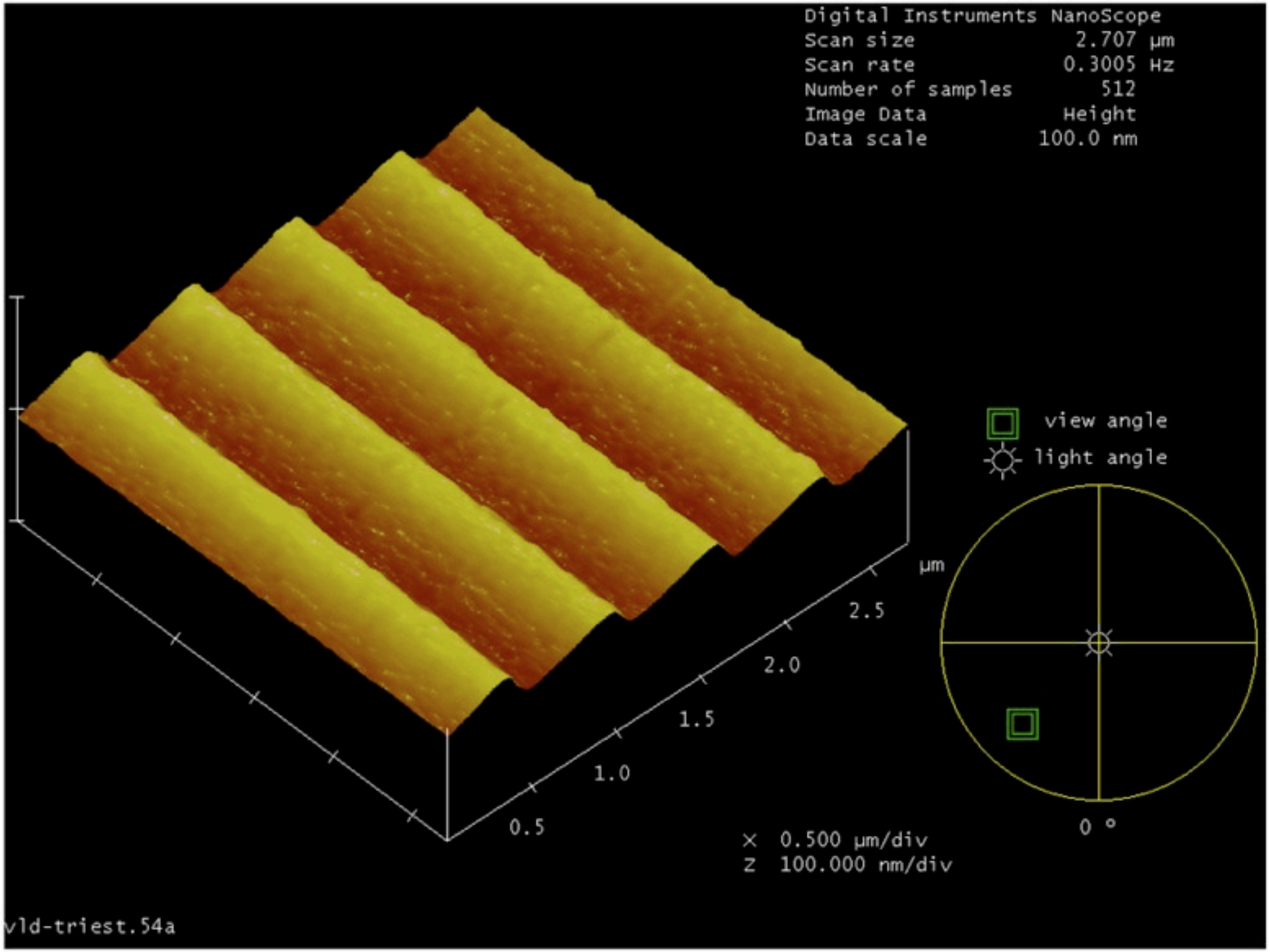
Laminar profile



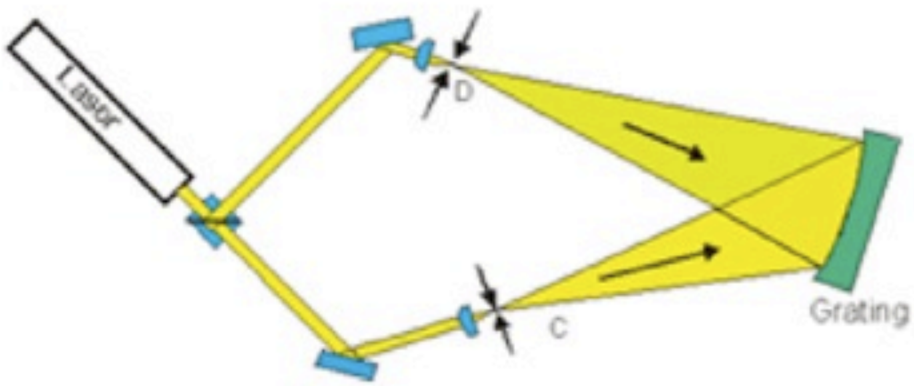


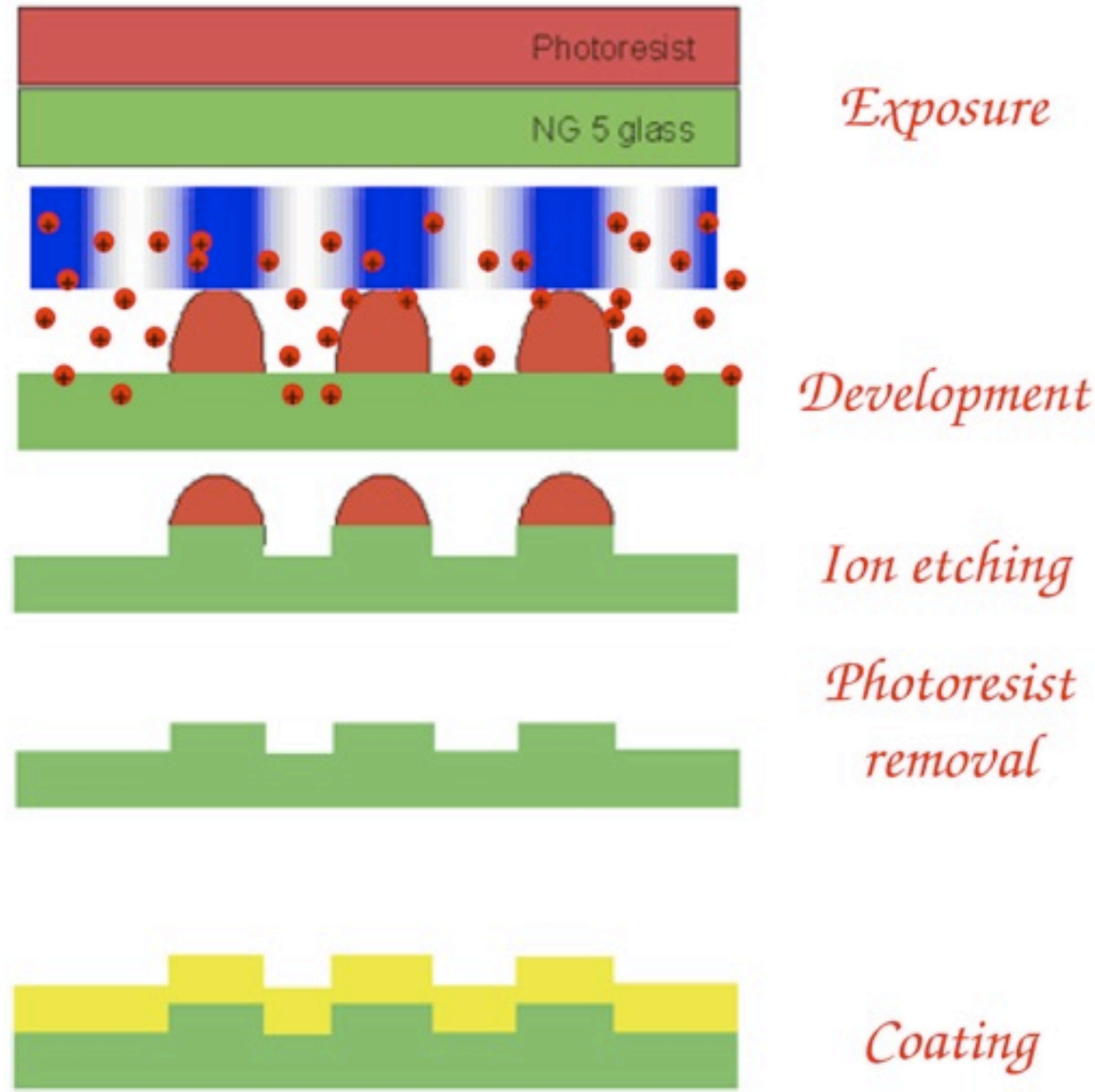
Mechanically ruled grating

Mechanically ruled (CARL ZEISS Grating Ruling Engine GTM6) with blazed profile down to 0.5-0.7°

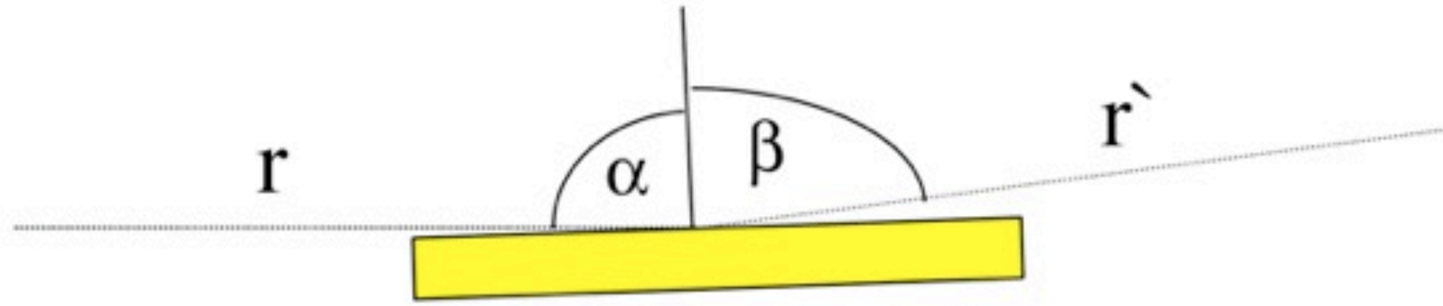


Holographical grating





Grating's equations



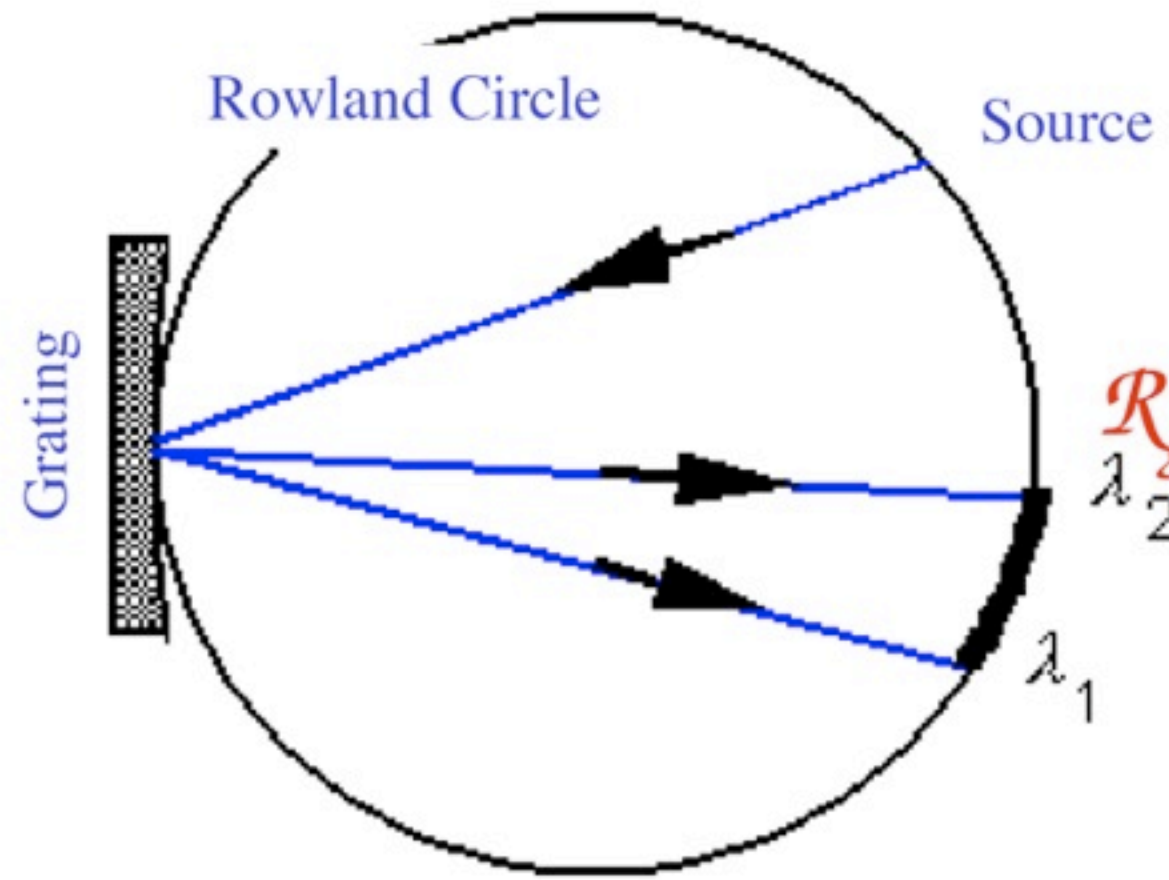
Optical path function

$$F_{100} = -n\lambda D_0 + (\sin\alpha - \sin\beta)$$ *grating equation*

$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right)$$ *tangential focus*

$$F_{300} = \left[\left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} \right) \frac{\sin\alpha}{r} + \left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right) \frac{\sin\beta}{r'} \right]$$ *primary coma*

Rowland condition



$\mathcal{F}_{200}=\mathcal{F}_{300}=0$

$r=\mathcal{R}\cos\alpha$

$r'=\mathcal{R}\cos\beta$

$\mathcal{R}_{grating}=2\mathcal{R}_{rowland\ circle}$

$$F_{200}=\left(\frac{\cos^2\alpha}{r}-\frac{\cos\alpha}{R}\right)+\left(\frac{\cos^2\beta}{r'}-\frac{\cos\beta}{R}\right)$$

tangential focus

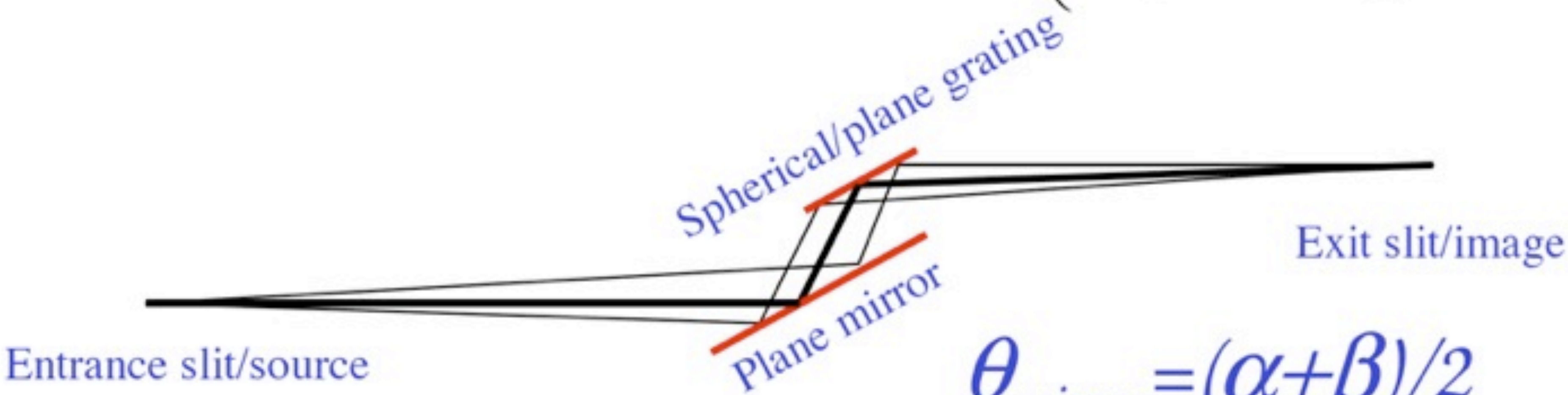
$$F_{300}=\left[\left(\frac{\cos^2\alpha}{r}-\frac{\cos\alpha}{R}\right)\frac{\sin\alpha}{r}+\left(\frac{\cos^2\beta}{r'}-\frac{\cos\beta}{R}\right)\frac{\sin\beta}{r'}\right]$$

primary coma

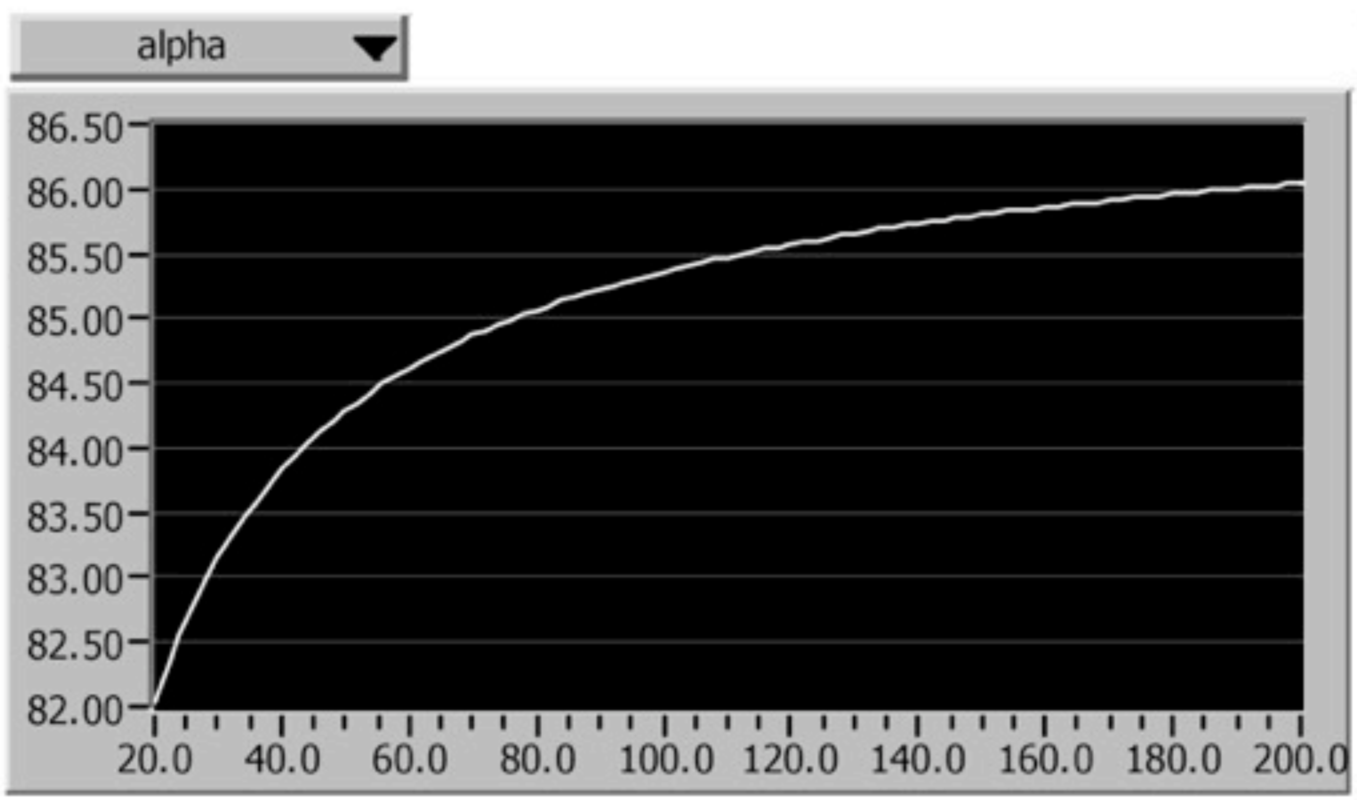
Plane/spherical grating monochromators

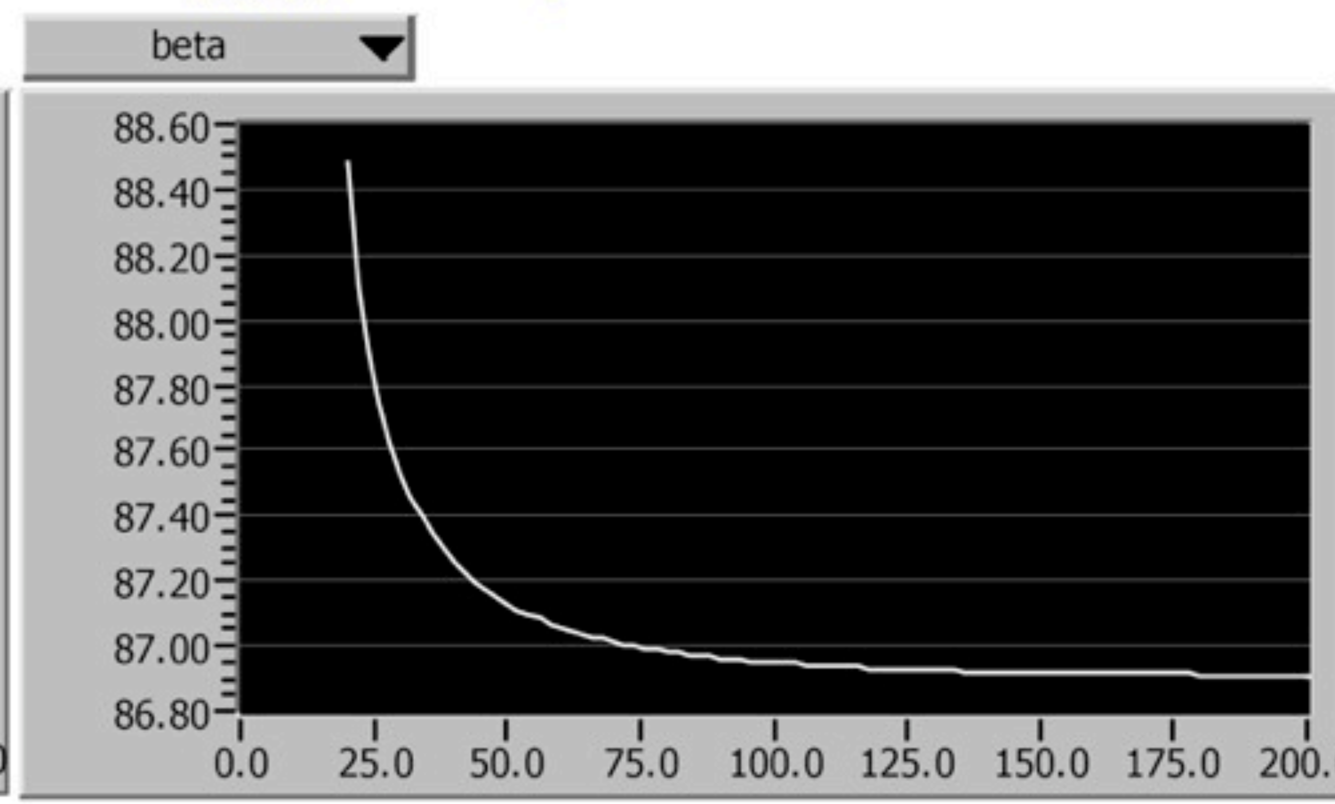
$$F_{100} = -n\lambda D_0 + (\sin\alpha - \sin\beta)$$

$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R}\right)$$

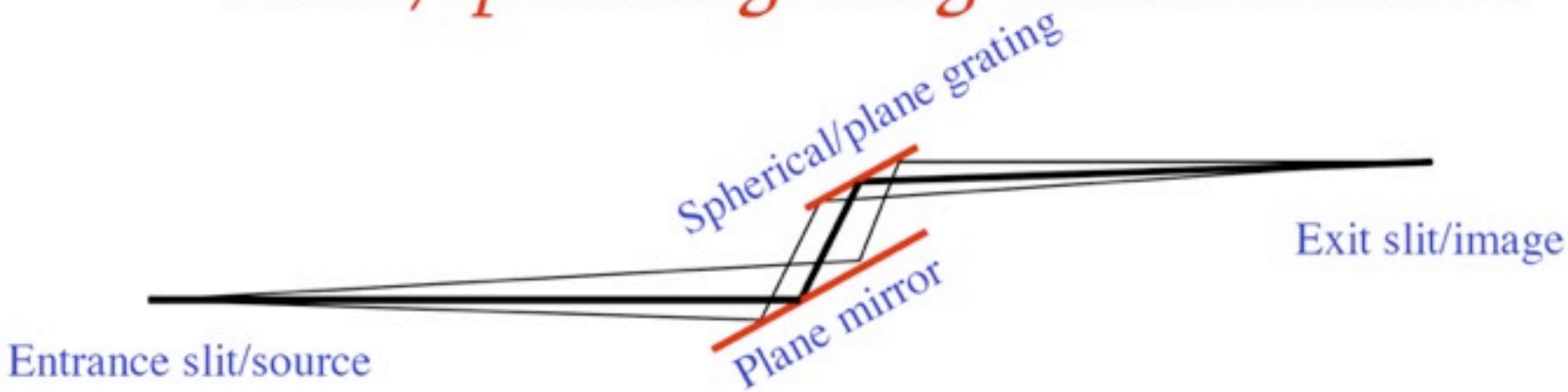


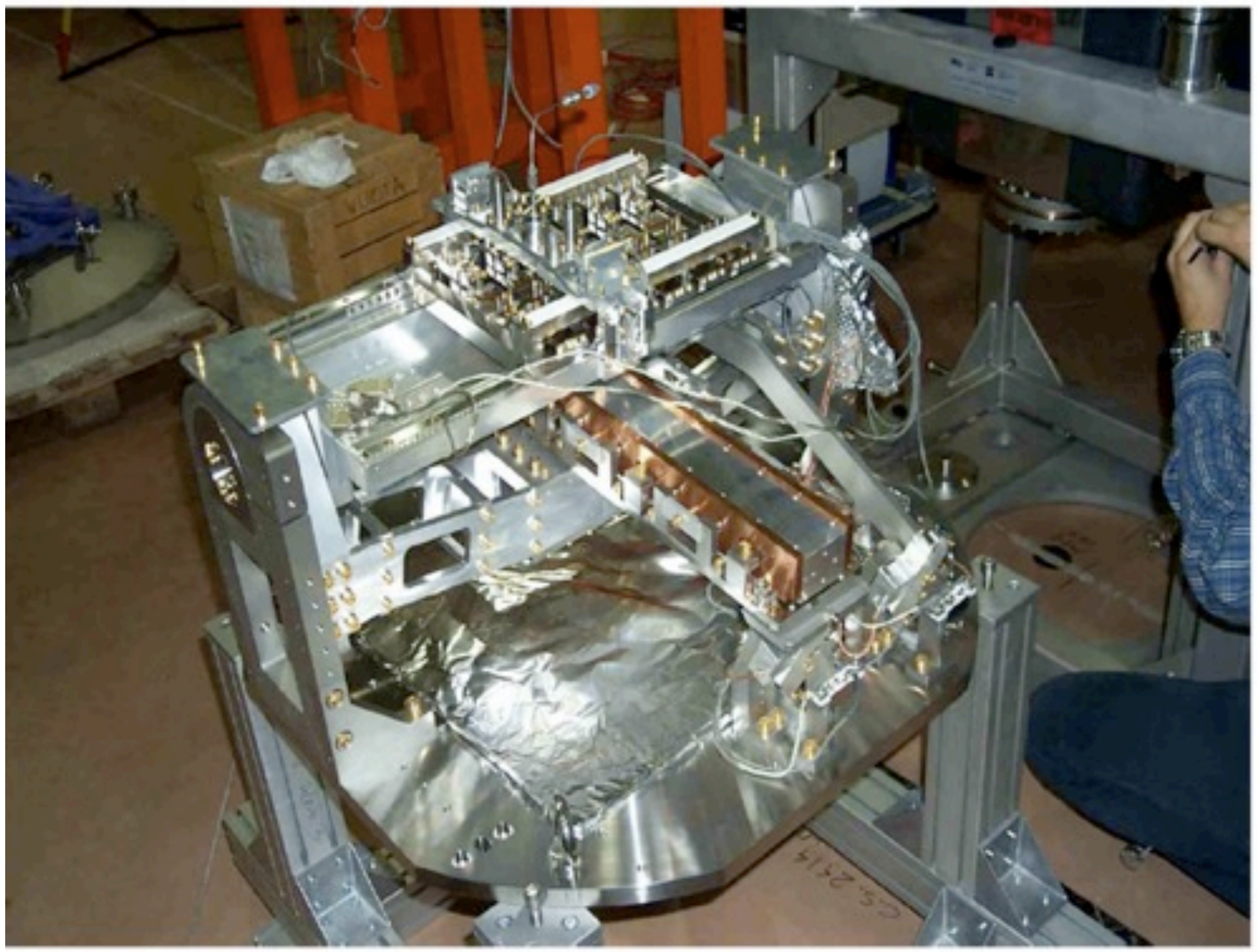
$$\theta_{mirror} = (\alpha+\beta)/2$$

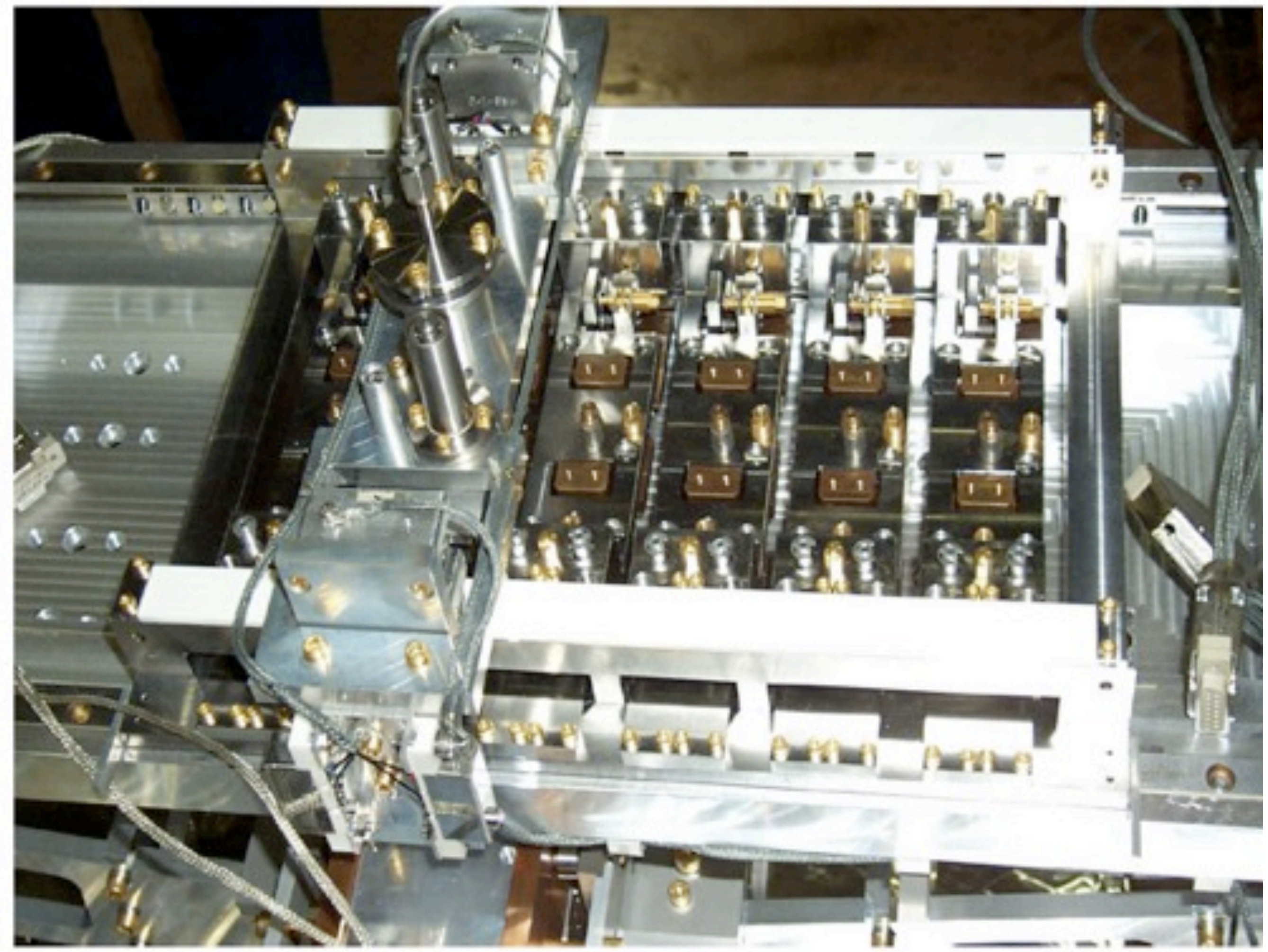




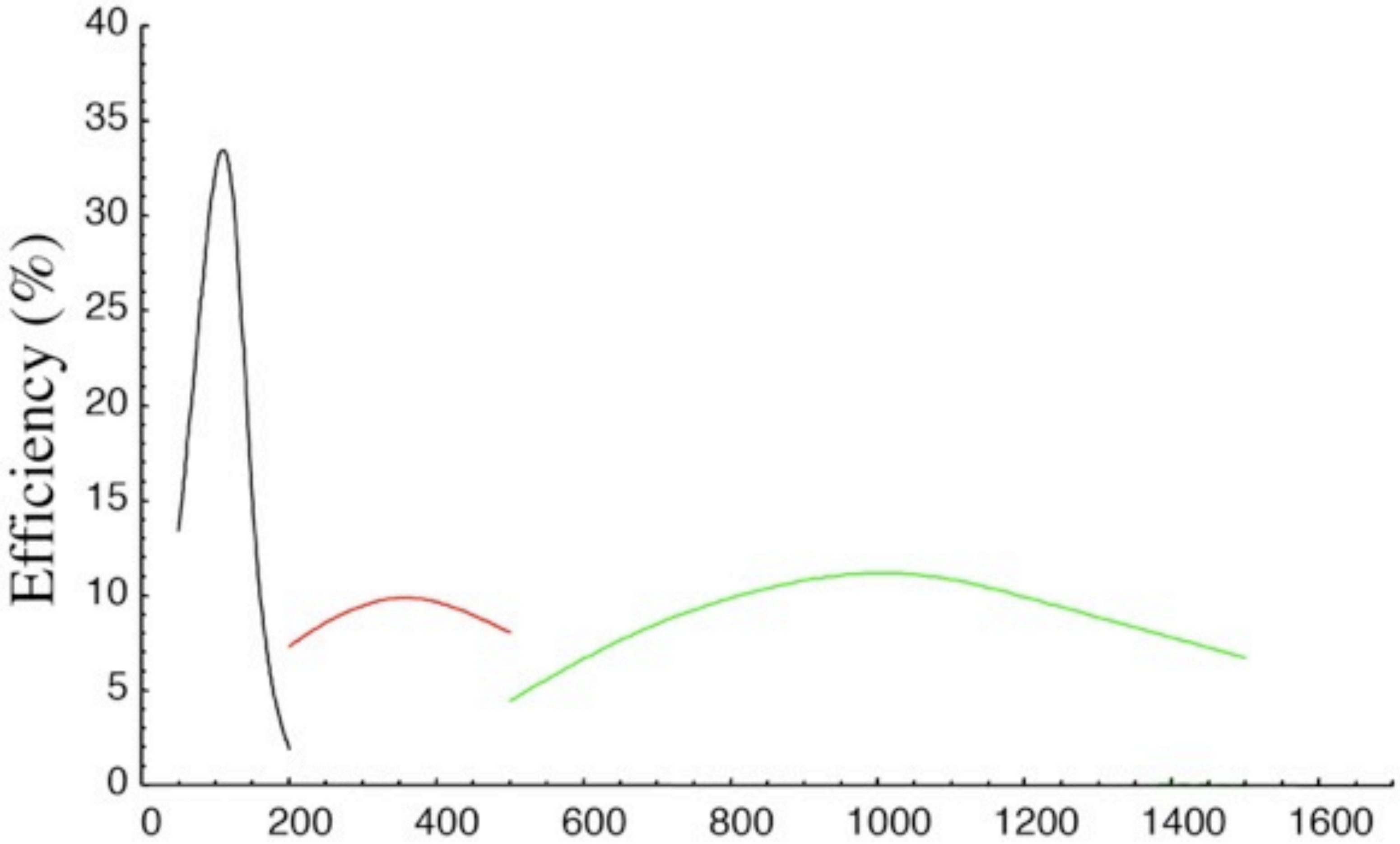
Plane/spherical grating monochromators

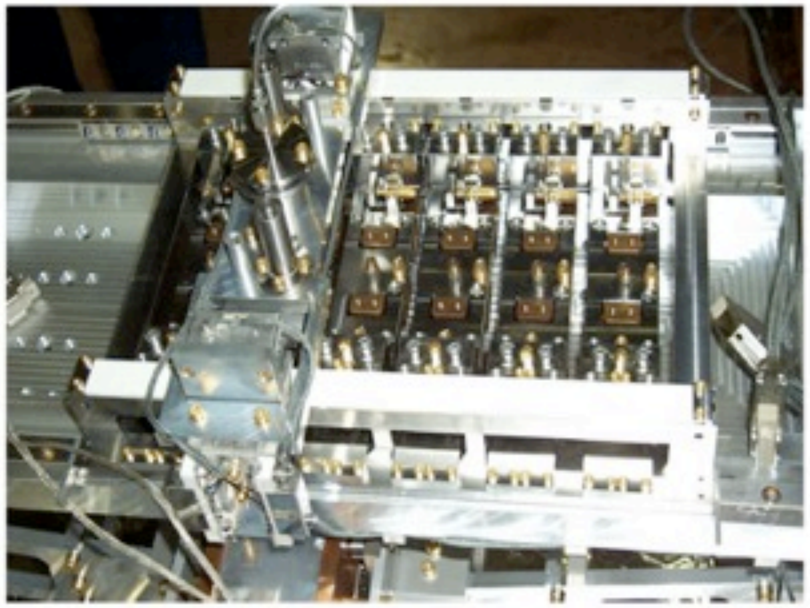


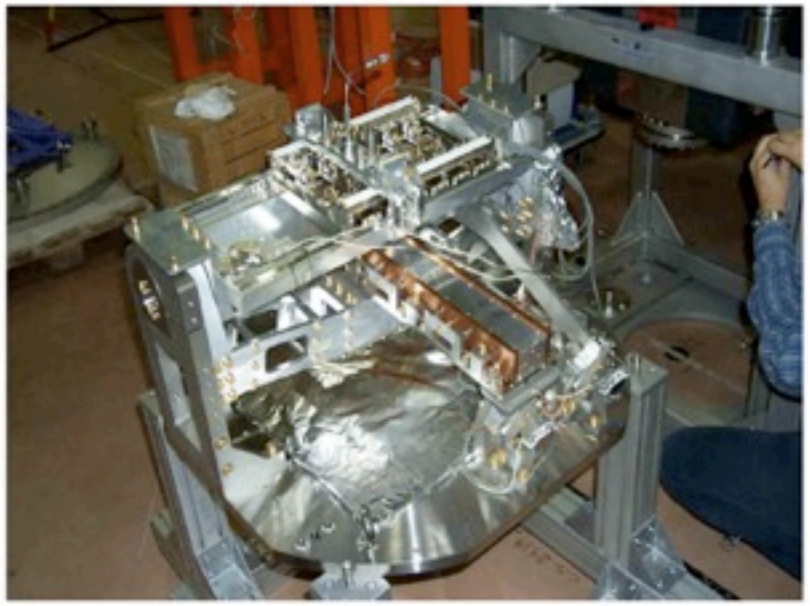




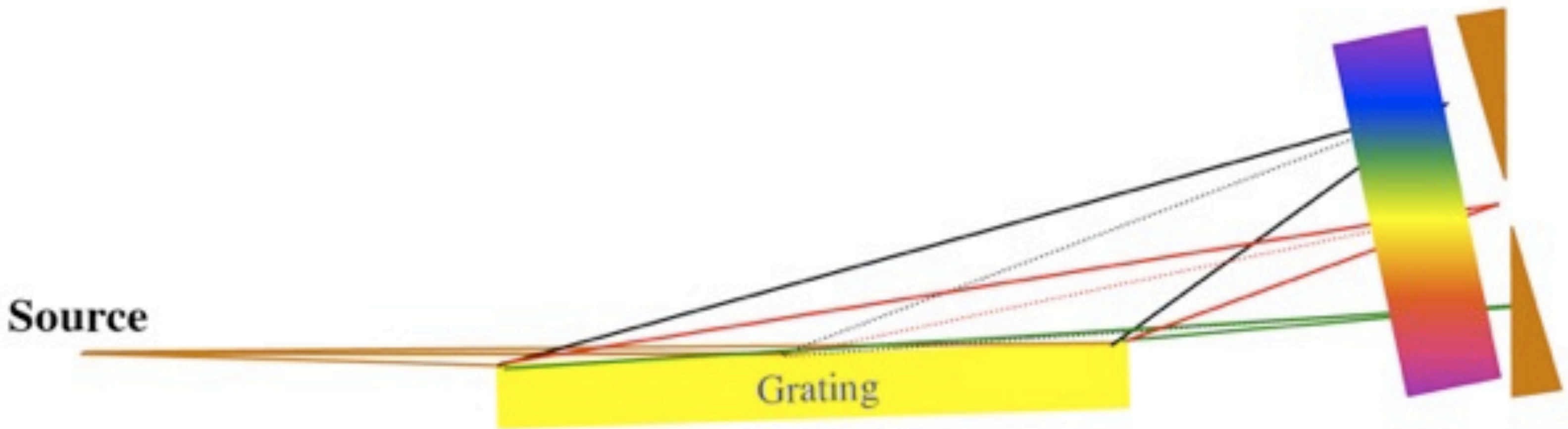
Spherical grating monochromator efficiency

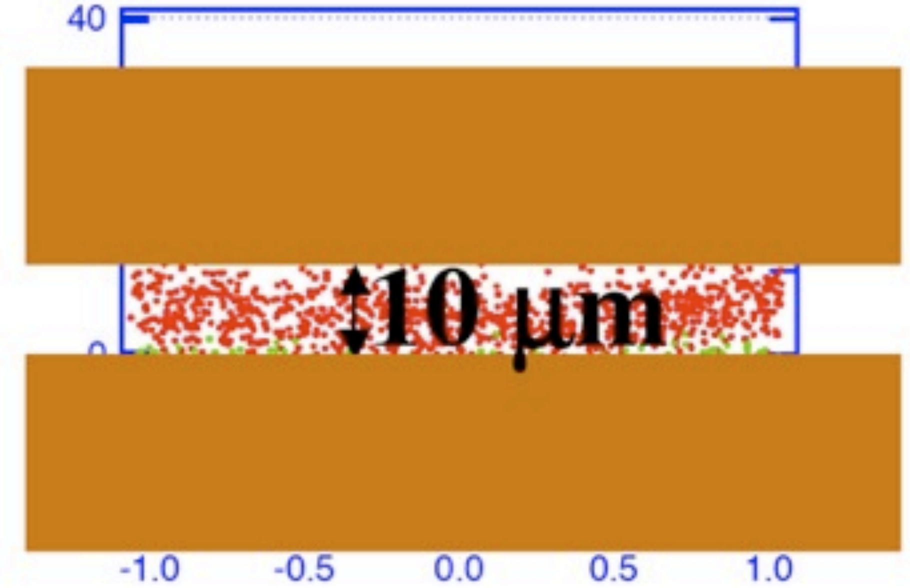




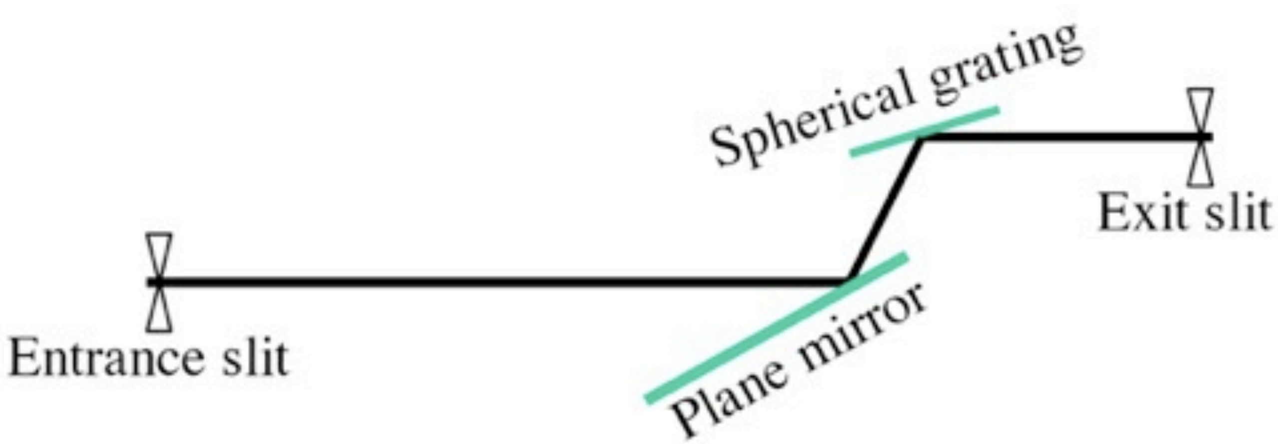


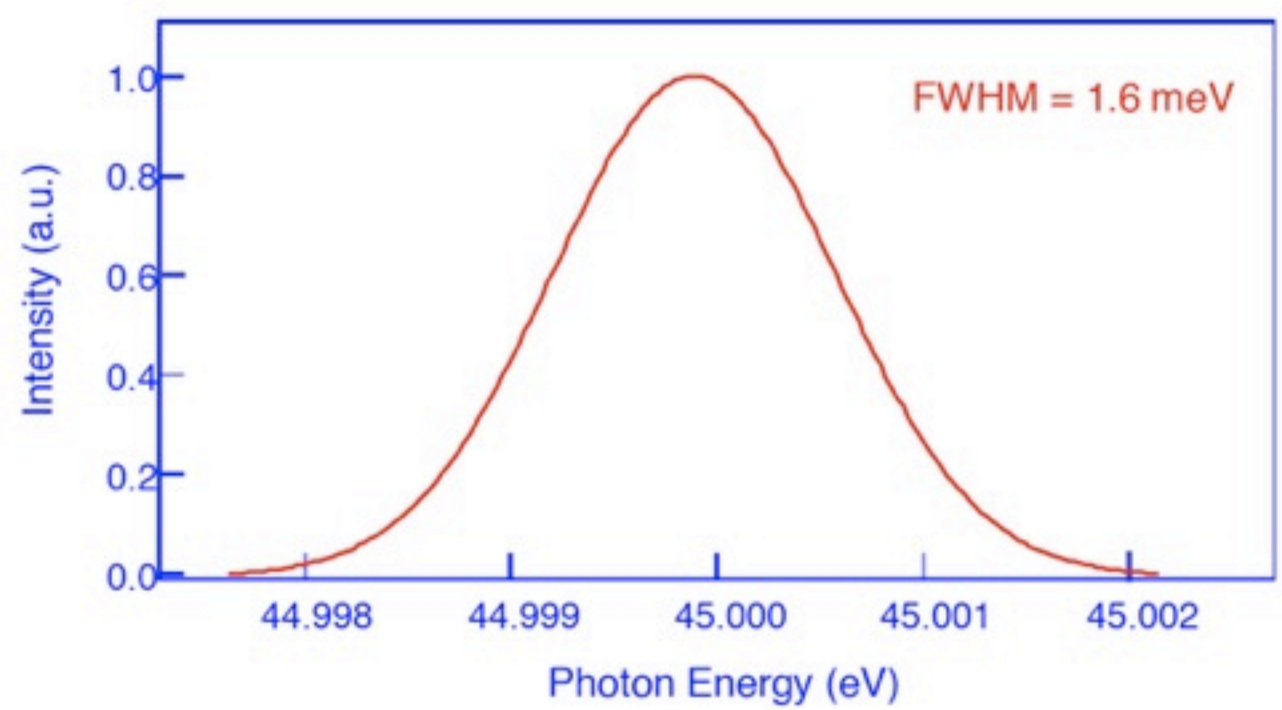
Resolving power





Exit slit





Resolving power

smaller are s and s´,
smaller will be the bandpass

$$Nk\lambda = \sin(\alpha) - \sin(\beta)$$

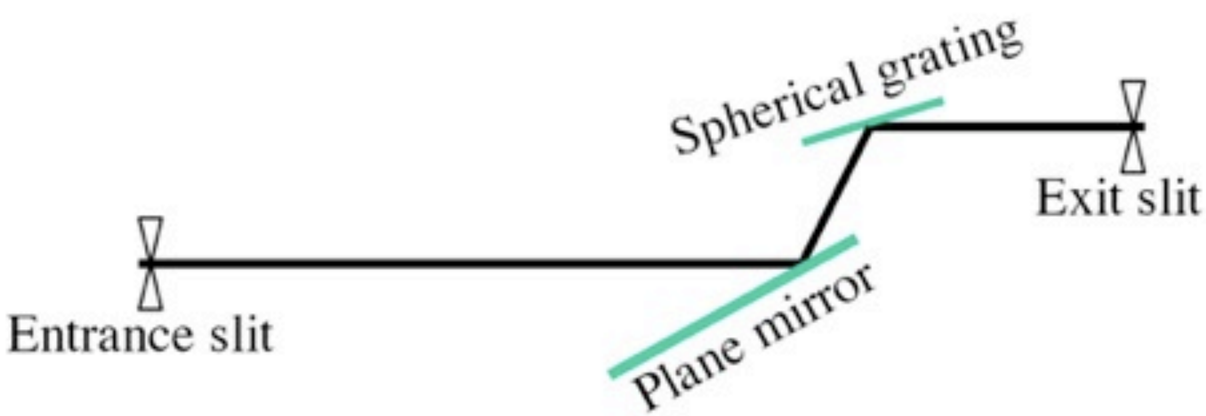
$$\left(\frac{\partial\lambda}{\partial\alpha}\right) = \frac{\cos(\alpha)}{Nk} \quad \Delta\alpha = \frac{s}{r} \qquad \Delta\lambda_{entrance} = \frac{s\cdot\cos(\alpha)}{Nkr}$$

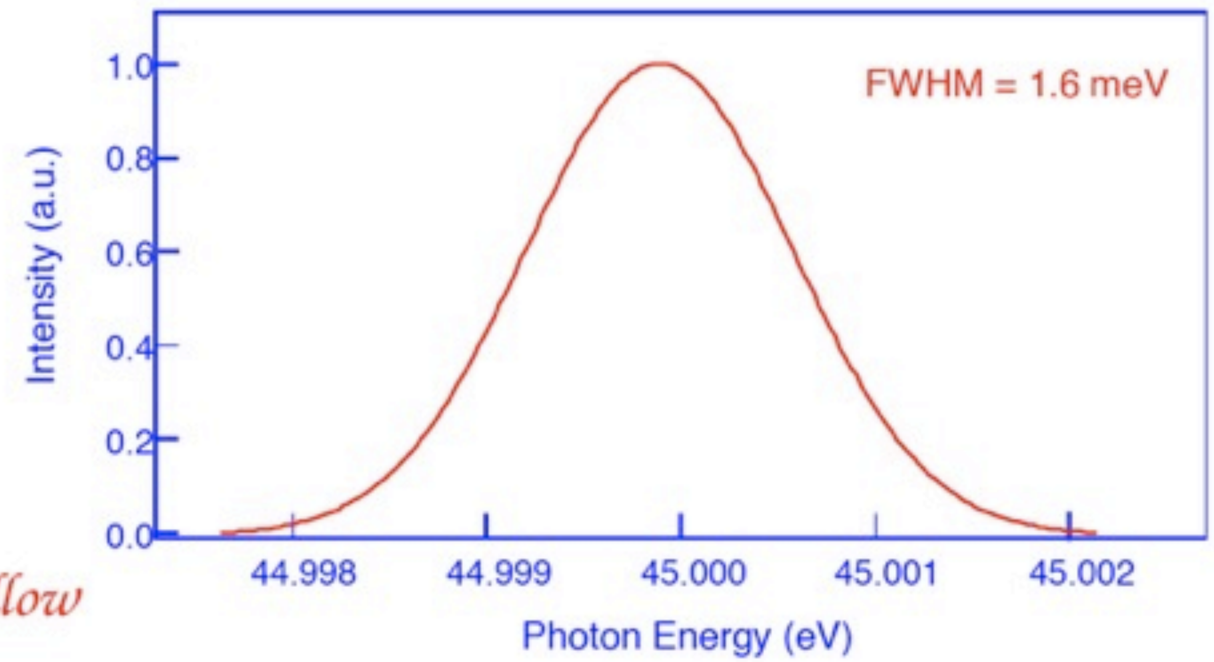
entrance slit contribution

$$\left(\frac{\partial\lambda}{\partial\beta}\right) = \frac{\cos(\beta)}{Nk} \quad \Delta\beta = \frac{s'}{r'} \qquad \Delta\lambda_{exit} = \frac{s'\cdot\cos(\beta)}{Nkr'}$$

exit slit contribution

Resolving power = E/ΔE



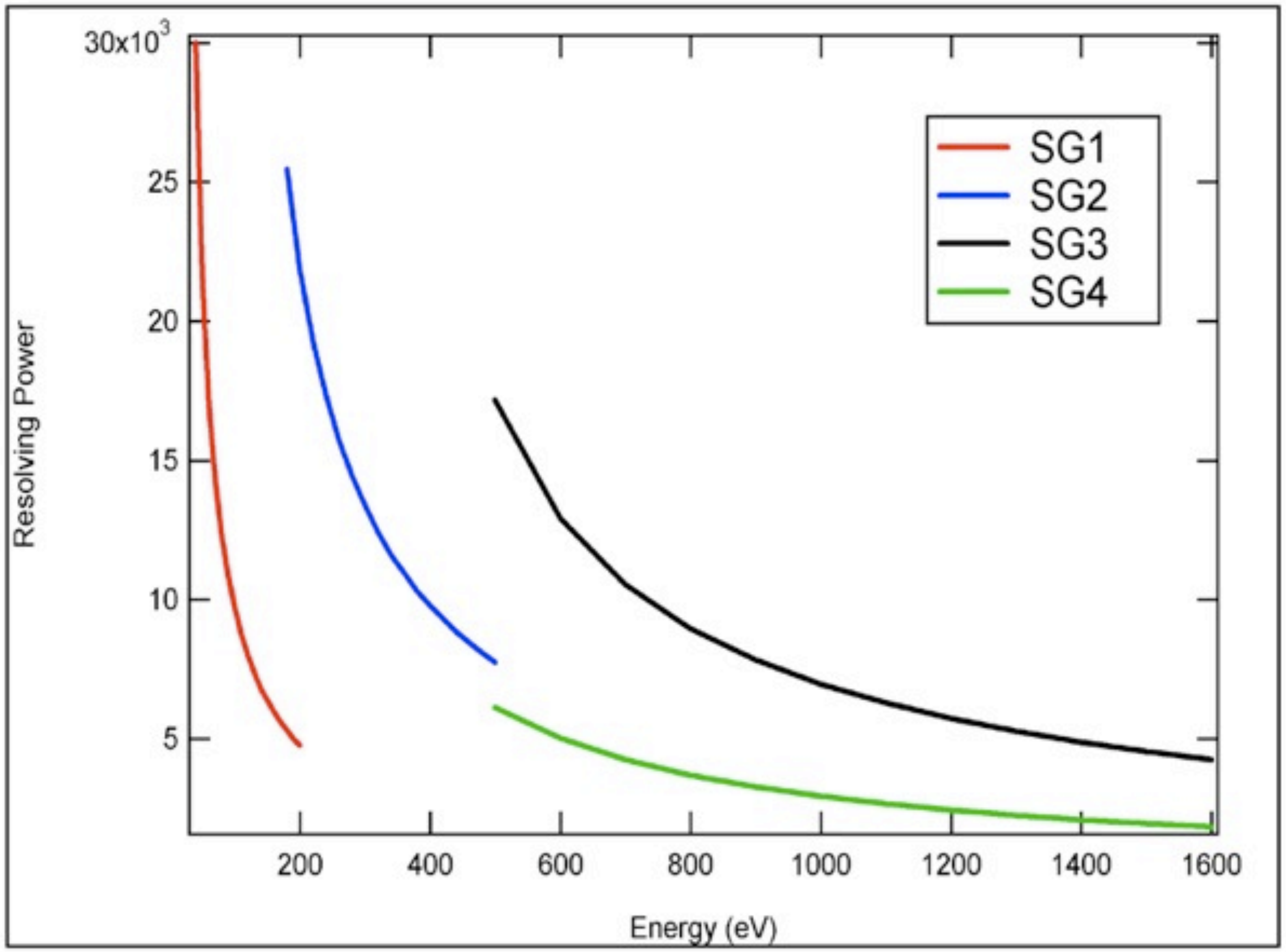


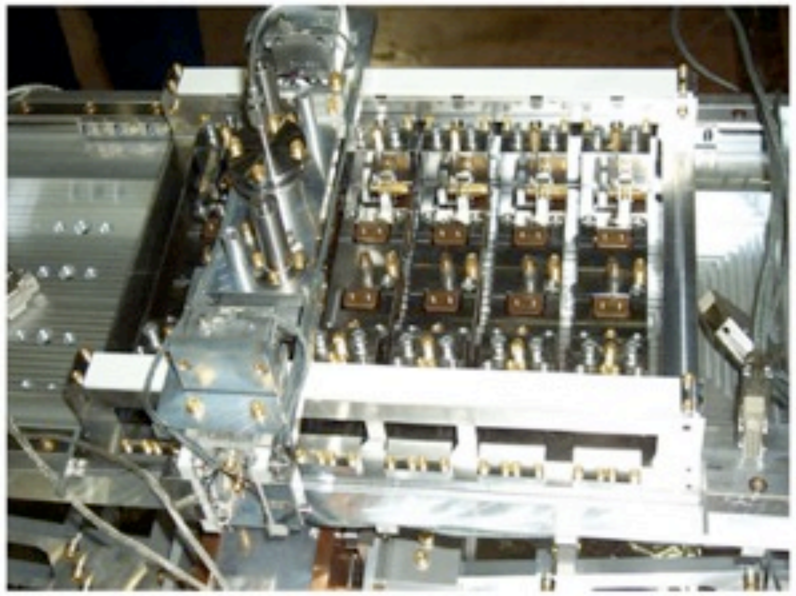
Note: the angular precision of the grating rotation have to allow the energy selection i.e. 0.2-0.3 μrad

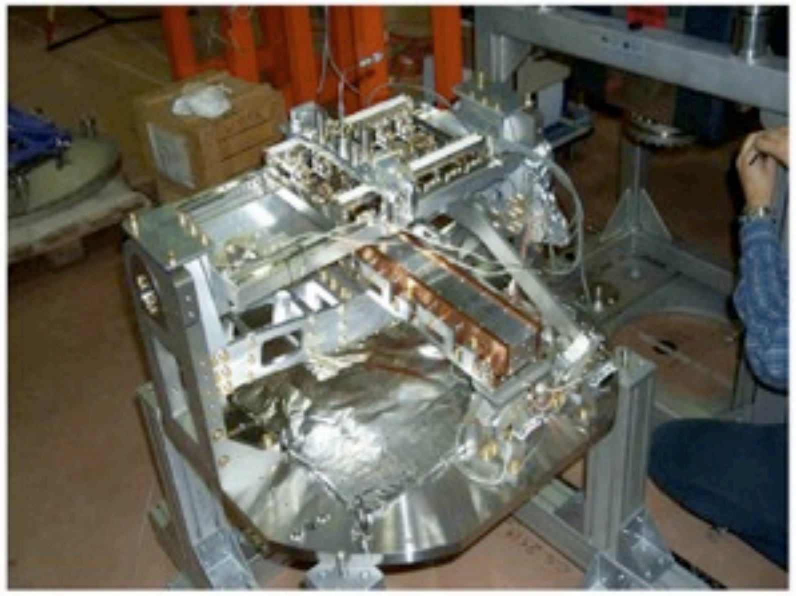
$45/0.0016 \approx 28000$

Resolving power

Typical Spherical grating monochromator resolving power







Resolving power