STABILITY OF A SHORT RAYLEIGH RANGE LASER RESONATOR WITH MISALIGNED OR DISTORTED MIRRORS

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Abstract

Motivated by the prospect of constructing an FEL with short Rayleigh length in a high-vibration environment, we have studied the effect of mirror vibration and distortion on the behavior of the fundamental optical mode of a cold-cavity resonator. A tilt or transverse shift of a mirror causes the optical mode to rock sinusoidally about the original resonator axis. A longitudinal mirror shift or a change in the mirror’s radius of curvature causes the beam diameter at a mirror to dilate and contract with successive impacts. Results from both ray-tracing techniques and wavefront propagation simulations are in excellent agreement.

INTRODUCTION

Some designs for a high-power free electron laser (FEL) call for a short Rayleigh length optical resonator in order to reduce the system size while minimizing heat damage to the mirrors [1, 2]. An additional advantage of this design is improved optical beam quality, due to the small interaction region in the center of the resonator [3]. However, this design raises concerns about mode stability, in particular the sensitivity to motions of the mirrors. This paper presents a study of the effect on beam behavior of mirror motion and mirror radius change, particularly as they affect short Rayleigh length resonators.

We study the results of several cavity distortions: mirror tilt, transverse and longitudinal shifts in mirror position, and changes in mirror focal length. In order to isolate resonator effects, our results are for a resonator alone with no gain. Since mirror motions are relatively slow (~ms) compared with the optical round trip time (~ns), the motions are assumed to be fixed over many passes of the beam through the resonator.

In general, the optical beam in a laser resonator retraces itself — it is an eigenmode of the resonator. If a mirror is misaligned or distorted, however, the resonator eigenmode will be redefined and the existing optical beam will tend to walk around the mirrors [4]. For sufficiently large misalignment, the beam radius may increase indefinitely — i.e., the resonator may become unstable. These effects are most pronounced for short Rayleigh length resonators, which are already near the stability limit. In practical terms, the mirror misalignment and distortion will cause the beam displacement to exceed the size of the mirrors, thereby creating beam loss and lowering the resonator Q.

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SIMULATION TECHNIQUES

We start by assuming a resonator with two identical mirrors of radius of curvature R (focal length f = R/2) separated by distance S and enclosing a Gaussian beam which is an eigenmode of the resonator with Rayleigh length z0 (Fig. 1). If we normalize all longitudinal distances by S, all transverse distances by (λS/πf) and all angles by (λ/πS)1/2, then f = z02 + 1/4, and the 1/e radius of the beam at any z is w(z) = (z0 + z2/z0)1/2. In particular, the waist radius is w0 = z01/2 [5]. For a 10 m long resonator with λ = 1μm, the transverse scaling length is 1.8 mm and the scaling angle is 0.18 mrad.

Figure 1: Resonator with Gaussian mode characterized by Rayleigh length z0. Distortions of the right-hand mirror include tilt θm, transverse shift h, longitudinal shift ΔS, and focal length change Δf (not shown).

The beam is simulated using two techniques. In the ray tracing technique, a Gaussian beam is simulated by a random collection of rays, Gaussian distributed in both transverse position y and angle θ and set up at the beam waist [6]. For a beam whose amplitude in the y-plane is

\[ A \exp(-y^2/w_0^2) \]

the joint probability density is given by

\[ f(y, θ) = \frac{1}{π} e^{-(y^2+z_0^2θ^2)/z_0}, \]

(1)

Here z0 is the angular spread of the beam at z >> z0 as shown in Fig. 1. Each ray is then propagated numerically with the usual ABCD ray matrices and the evolving ray density and direction is found to closely emulate the actual behavior of a Gaussian beam.

In the wave propagation technique [1], the spatial part of the Gaussian beam \( a(x,y,z) \) is set up at the beam waist and then propagated numerically by the paraxial wave equation

\[ \partial_z a = (-i/4) \nabla_x^2 a \]

Both simulation methods can accommodate tilted,
shifted, and distorted mirrors, while the latter method can also incorporate laser gain.

In a third analytical method [5], the Gaussian beam is represented by complex beam radius \( q(z) = z + iz_0 \). Propagation is then accomplished using the ABCD matrix elements in the form \( q_2 = (Aq_1 + B)/(Cq_1 + D) \) and extracting the beam front curvature \( R(z) \) and beam radius \( w(z) \) from \( 1/q = 1/R - i/w^2 \). This method will accommodate longitudinal mirror shift and focal length change only. However, when coupled with ray tracing, can also describe the effects of tilt and transverse shift of the mirrors.

**MIRROR TILT AND SHIFT**

We now let the right-hand mirror undergo tilt \( \theta_m \) and/or transverse shift \( h \) and investigate the subsequent behavior of the Gaussian beam. The immediate effect is that the reflection angle of any ray incident on the mirror will be increased by \( 2\theta_m h/f \). The resonator will remain stable, but a new resonator axis will be defined which tilts with respect to the old axis by amount \( \phi \), where

\[
\phi = -\left[ (1 + 4z_0^2)\theta_m + 2h \right]/(8z_0^2).
\]

The optical beam, which initially was an eigenmode of the old resonator, now becomes tilted with respect to the new axis and is no longer an eigenmode of the realigned resonator. Consequently, with each reflection, its angle with respect to the old axis will change in a rocking fashion, depending on the value of \( z_0 \).

The effect of the rocking over many passes \( n \) is to make the beam position on the mirror walk sinusoidally up and down. If \( y_n \) is the beam position on the mirror after \( n \) reflections [5],

\[
y_n = C_1[1 - \cos(\alpha n)] + C_2 \sin(\alpha n).
\]

where

\[
\alpha = \cos^{-1}\left( \frac{2f^2 - 4f + 1}{2f^2} \right),
\]

For the small \( z_0 \) case we are concerned with here, \( y_c \) is a strong function of \( z_0 \). We show this dependence in Fig. 3 where \( y_c/\theta_m \) and \( y_c/h \) are plotted separately against \( z_0 \). As \( z_0 \) becomes smaller, the transverse excursions become comparable to the mirror diameter and the beam will walk off the mirrors.

**LONGITUDINAL MIRROR SHIFT**

Let the resonator contain a Gaussian beam which is a resonator eigenmode with Rayleigh length \( z_0 \). Since \( z_0 \) is small, the mirror focal lengths \( f = z_0^2/2 + 1/4 \) are already only slightly larger than the resonator stability limit \( f_{\text{min}} = 1/4 \). Let the right-hand mirror shift by \( \Delta S \) in the \( z \)-direction. Successive reflections of the beam will remain on axis, but the Rayleigh lengths of the beam and the resonator eigenmode will no longer be equal. If \( \Delta S \) is positive (cavity length increases), the focal lengths decrease to \( f' = f/(1 + \Delta S) \), and if \( f' < 1/4 \),...
Figure 4: Evolution of an optical beam in a resonator with $z_0 = 0.1$ and right mirror shift $\Delta S/S = 0.031$. The vertical lines represent mirrors, with successive reflections unfolded to see the overall behavior. The gray areas are the trajectories of 1000 random rays; the dotted lines, calculated from beam theory, correspond to the radius $w(z)$ of the Gaussian mode. The beam remains on axis, but expands and contracts with successive reflections.

The resonator will become unstable and the beam will expand without limit. The maximum allowable value for $\Delta S$ is therefore $\Delta S_{\text{max}} = 4f - 1 = 4z_0^2$.

If $\Delta S < \Delta S_{\text{max}}$, the resonator remains stable but the beam will no longer retrace itself in succeeding passes, as shown in Fig. 4. With each pass, the beam width at the mirrors will expand and contract, depending on both $\Delta S$ and $z_0$. Figure 5 shows the effect of varying $\Delta S$ for several $z_0$. For $\Delta S < \Delta S_{\text{max}}$, the effect on $y_{\text{max}}$ is small. However, as $\Delta S$ approaches $\Delta S_{\text{max}}$, $y_{\text{max}}$ increases and finally diverges at $\Delta S_{\text{max}}$.

For $\Delta S < 0$ (resonator length decreases), the resonator remains stable but the beam width again expands dramatically as the difference between the Rayleigh lengths of the beam and the resonator eigenmode becomes large.

Figure 6: Evolution of an optical beam in a resonator with $z_0 = 0.1$ and right mirror shift $\Delta f/f = -0.05$.

MIRROR DISTORTION

Now let the focal length $f$ of the right-hand mirror in the previously undistorted resonator change by amount $\Delta f/f$. Since the mirror focal lengths are unequal, the mode waist of the resonator eigenmode will move away from the resonator center. The effect is to change the resonator eigenmode so that it no longer corresponds to the original beam. Consequently the beam radius on the mirror will expand and contract with each subsequent reflection, as shown in Fig. 6. In addition, if $\Delta f/f$ is negative (a decrease in the mirror focal length) and made too large, the resonator will no longer be stable and the beam will diverge indefinitely. The stability criterion is $\Delta f > -8z_0^2/(1 + 4z_0^2)$.

Figure 7 shows the results from our simulations. The beam radius at the mirror is $y_{\text{max}}$, as before. As $\Delta f$ is made increasingly negative, $y_{\text{max}}$ increases slowly as the threshold for resonator instability (vertical dashed lines) is approached, and then diverges sharply at the threshold.

DISCUSSION

We have shown that for a short Rayleigh length resonator with no gain, the effects of mirror tilt, shift, and focal length change are significant. For $\Delta S < \Delta S_{\text{max}}$, the effect on $y_{\text{max}}$ is small. However, as $\Delta S$ approaches $\Delta S_{\text{max}}$, $y_{\text{max}}$ increases and finally diverges at $\Delta S_{\text{max}}$.

Figure 5: Maximum beam radius $y_{\text{max}}$ for right-hand mirror shift $\Delta S$ at several values of $z_0$. The vertical dashed lines show the limits of resonator stability at $\Delta S_{\text{max}} = 4z_0^2$. The data points are taken from ray and beam simulations; the solid lines are guides to the eye. For an FEL with $S = 10$ m and $\lambda = 1$ $\mu$m, $y_{\text{max}} = 10$ corresponds to 1.8 cm.

Figure 6: Evolution of an optical beam in a resonator with $z_0 = 0.1$ and right mirror shift $\Delta f/f = -0.05$.

Figure 7: Maximum beam radius $y_{\text{max}}$ for focal length change $\Delta f/f$ of the right-hand mirror at several values of $z_0$. The minus sign in front of $\Delta f$ indicates the focal length is decreasing. The points are taken from ray simulations and beam calculations; the solid lines are guides to the eye; and the vertical dashed lines show the limits of resonator stability at $\Delta f = -8z_0^2/(1 + 4z_0^2)$. [Graphs and figures are not included in the text.]
change can produce dramatic changes in the beam direction and width. In the cases of mirror tilt and transverse shift, the effect is to cause the beam to rock up and down on the mirrors and, if the rocking amplitude is sufficiently large, to cause the beam position to exceed the mirror radius. In the cases of longitudinal mirror shift and focal length change, the beam will remain on axis but the beam radius at the mirror will expand and contract with successive reflections. If the beam radius becomes too large, portions of the beam may exceed the mirror radius. In either case beam power can be lost, or, equivalently, the cavity $Q$ will be reduced. For comparison with actual mirrors, the $y$-axes in Fig. 3, 5, and 6 can be converted to real values by multiplying by the transverse scaling length $(\lambda S/\pi)^{1/2}$. For a laser with $S = 10$ m and $\lambda = 1$ $\mu$m, $y = 10$ corresponds to an actual $y$ of 1.8 cm.

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