SPOT-TO-BEAM PROCEDURE

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Abstract

We describe the interactive spot-to-beam MATHEMATICA procedure for a) approximating the spot image at the screen (and beam at screen position) as an ellipse, b) getting five parameters of the elliptic beam (two diameters, center coordinates, and orientation angle). The basic idea is to "map" the *reference* holes at the diagnostic screen onto the XY plane normal to the beam propagation direction (Z-axis). All distortions of the image, e.g., due to camera-screen disposition can be, in principle, taken into account. With the non-linear LSM fitting, the "curved" coordinate system of the holes at image is transferred to the Cartesian "Laboratory" coordinate system (C.S.) at XY plane. Then the fitting ellipse is found in the C.S., by solving the system of N linear equations for 5 unknown parameters of beam ellipse, where N>5 is a number of the sample points on edge (boundary) of the spot image. Examples of the real measurements in the Israeli Electrostatic Accelerator FEL (EAFEL) are demonstrated. The accuracy of the beam diameter values is ≈.5 mm depending on picture quality and the operator's experience (and patience!). The procedure is to be used in routine measurements of EAFEL to improve the electron beam transport.

INTRODUCTION

In the EAFEL [1], the distance between the electron gun cathode and the accelerator's entrance is about 1.9 m and 45 keV, 2 Amp electron beam is transported through this space by using (among other electron-optics elements) four focusing coils C1-C4 and two diagnostic screens SP and S0. Unfortunately the hardware and software used by us at present do not provide a brightness distribution in a spot image and even the spot boundary is not always well defined. Hence we had to try to measure the spot (and beam) geometric parameters to the best of our ability.

EXPERIMENTAL LAYOUT

The scheme of the focusing coils *C1-C4* and the diagnostic screens SP and S0 is shown in Fig. 1.

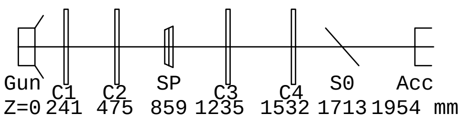

Figure 1: Experimental layout (not in scale).

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We use the following numerical values of parameters: $E = 45$ keV, electron beam energy; Z-coordinates of screens and coils (assuming that electron gun is at Z=0): $z_{sp} = 859$ mm, position of pepperpot screen SP (Ti-Al_2O_3), $z_{s0} = 1713$ mm, position of S0 screen (ceramic); $z_1 = 241$ mm, $z_2 = 475$ mm, $z_3 = 1235$ mm, and $z_4 = 1532$ mm, positions of focusing coils C1, C2, C3 and C4, respectively.

Note that SP screen is rotated by 45 deg around vertical (Y) axis, while S0 screen is rotated by 45 deg around horizontal (X) axis, both X and Y axes being normal to Z axis, direction of beam propagation. For the spot measurement purposes, there are made *reference holes* at all diagnostic screens of EAFEL. Due to the various optical distortions (mainly caused by too close and not proper camera-screen disposition), the problems with the frame grabber hardware and software etc., there is no direct way to measure the spot (and beam) parameters from the raw images. That is why the special interactive Spot-to-Beam (STB) procedure (written in MATHEMATICA [2]) was developed in the EAFEL group.

STB PROCEDURE

Basic assumptions of STB

The basic assumptions of the STB procedure are:

a) the *reference hole* positions at the diagnostic screen are well known and their centers are perfectly aligned along straight lines (that is we ignore manufacture errors if any)

b) the screen position is known (the screen center is on Z-axis which is *not* necessarily position of the spot and beam centers), the screen rotation angle is 45 deg, and only around one of axes, X or Y

c) we neglect the difference in the ray paths to the nearest and farthest sides of screen (this difference is $D/\sqrt{2} \approx 0.71\,D \approx 3.6$ cm for the screen diameter $D = 5$ cm); we do such not because we think that this difference is negligible but rather because right now we do not know how properly to take it into account in the STB procedure

d) the other essential assumption is that the beam (necessarily) and the spot (optionally) can be described as the elliptical figures

e) it is assumed that the beam propagates (at least near the screens) along the Z-axis; otherwise the procedure gives the parameters of the cross-section of the elliptic beam normal to Z-axis (that is the real dimension of the elliptic beam is *smaller* than that given by the procedure).

The "flow-chart" of STB

The STB procedure of the spot/beam treatment is highly interactive: it requires the experimentalist/operator's active involvement. If the specialized hard/sofware is available the whole procedure can be fully automated.

At present, the STB procedure comprise of the following stages.

Reference picture of the screen First we need the reference picture of the diagnostic screen with the best seen reference holes. By using any graphical software we save the reference file in any suitable format.

Spot image at diagnostic screen We need the picture of spot image at diagnostic screen with all necessary data, focusing/steering coil currents, quad currents etc. If (which is most preferable) the image of spot with clearly seen reference holes is available, the "reference picture" is not necessary. Also we note that the reference picture and the spot picture should be got for the same camera-screen disposition etc. Again we save the spot image file in any suitable format.

Import the image file STB procedure itself starts with using MATHEMATICA's "Import" command to input the reference and spot image files (in any format) into MATHEMATICA notebook.

Getting coordinates of reference holes Then by using "Get Graphic Coordinates" command we obtain coordinates of the reference holes (in the coordinate system of the imported image), and save them as the **"holes"** variable with 2 x N_x x N_y elements (each of them being 2-vector, $\{x_i,\, y_i\}$ of i-th hole), where N_x is the number of holes in each x-row, and N_y is the number of holes in each y-column (for rectangular array of reference holes, otherwise the procedure is slightly more complex). For example, at SP screen, $N_x = N_y = 5$, while at S0 screen, $N_x = 3$ and $N_y = 5$.

Getting coordinates of spot boundary Similarly, the coordinates of the N (N should preferably be $\gg$5) "sample" points at the spot image boundary are got and saved in the **"spot"** variable with 2xN elements (each of them being 2-vector, $\{x_i,\, y_i\}$ of i-th spot boundary point).

This finishes the "data extracting" procedure and the further treatment operations are simply done by "clicking" the relevant cells in MATHEMATICA notebook with right sequence. Still some knowledge of "what is going on" is useful.

Mapping reference holes to the Laboratory C.S. The "holes" variable is used to "map" the holes (their coordinates, $(x_i,\, y_i)$), to the "Laboratory" Coordinate System (C.S.), (X,Y),: e.g. x[central hole]→X = 0, y[central hole]→Y = 0; x[next-to-central right hole]→ X (= 10 mm for S0 screen, and = $5/\sqrt{2}$ mm for SP screen), y[next-to-central upper hole]→ Y (= 5 mm for SP screen, and = $10/\sqrt{2}$ mm for S0 screen).

Very important is the scaling by $1/\sqrt{2}$ for x-coordinates at SP screen and y-coordinates at all other screens of EAFEL. At this stage it is assumed that the diagnostic screen is perfectly aligned with its central hole's center at Z-axis and the angle of the screen rotation is exactly 45 deg. Otherwise instead of $1/\sqrt{2}$ we should use $1/\cos\varphi$ where φ is an inclination angle. We mention that $\varphi = 45 \pm 1$ deg gives 2% relative deviation from $1/\sqrt{2}$. In contrary, if rotation (around second axis) is (erroneously) non-zero, the effect is much less: for $\varphi < 12$ deg the deviation from 1 is less than 2%.

Transition from image c.s. to Lab. C.S. Now using the fact that reference holes "are connected" to the laboratory coordinate system, we find the *transition rule* from the whole {x,y} c.s. of the given image to the laboratory {X,Y} C.S. To this end we solve by LSM (actually we use "Fit" command of MATHEMATICA), $N_y \cdot N_x$ non-linear equations, separately for X(x,y) and Y(x,y). In the simplest case we use only the linear and cross-terms in (x,y), that is e.g $X(x, y) = c_0 + c_1\, x + c_2\, y + c_3\, x\, y$ and the similar "rule" (with other coefficients of course) is for Y(x,y).

Transformation of reference holes and spot to Lab. C.S. Using the *transition rules* X(x,y) and Y(x,y) we transform the reference holes positions at image to points in Lab. C.S., and also do the same for sample points of spot boundary. We present, in Fig. 2, left panel , the example for the case of I4 = .8 A. In getting beam in Fig. 2, we used

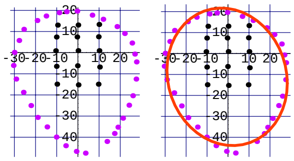

Figure 2: (Color) Left panel: the "true beam" and reference holes as obtained by STB for the case of I4 = .8 A. Right panel: additionally fitting ellipse found by STB is shown.

32 reference points. We may note that reference points are nicely aligning in the ellipse-like order. By checking such a figure one may notice the *outlying points* (which are most probably due to non-accurate clicking when getting coordinates of spot boundary). Then you may simply remove them from the **"spot"** variable, or try to obtain more accurate sample points.

This finishes the important part of the STB procedure: we obtained the "true beam"; and now we deal with another particular problem: how to draw ellipse through the given set of points and find the parameters of the ellipse.

FITTING ELLIPSE

This section describes another essential part of STB. We note that the problem of fitting the given set of point by any given curve is quite general and solved in many fields of science and technics. We only briefly outline the procedure used in STB.

Let us have N (in our particular case N=32) points with coordinates $\{x_i, y_i\}$, $i = 1, \ldots, N$. To fit elliptic curve through this set of points, we first write the general equation of the ellipse in the form

$$a\,x^2 + 2\,b\,x\,y + c\,y^2 + 2\,d\,x + 2\,e\,y + 1 = 0. \quad (1)$$

If i-th point lies exactly on the ellipse (1) then we may put $x \to x_i$ and $y \to y_i$ and Eq. (1) will be exactly valid. In general this equation will be valid only approximately, and we may ask which ellipse (that is which set of coefficients a, b, c, d, e) fits the given set of N points best. To find these coefficients we write the system of N equations

$$a\,x_i^2 + 2\,b\,x_i\,y_i + c\,y_i^2 + 2\,d\,x_i + 2\,e\,y_i + 1 = 0, \quad (2)$$

and the solve the system (2) by one of LSM methods. In MATHEMATICA the relevant command is

$$\{a, b, c, d, e\} = \text{PseudoInverse}\,[mat]\,.B, \quad (3)$$

where mat is the 5xN matrix of coefficients in system (2), and B is N-vector with all elements equal to -1.

Parameters of fitting ellipse

Now when we have coefficients of the ellipse equation, we can define parameters of ellipse.

First we write three so-called invariants of the ellipse

$$\triangle = \det \begin{pmatrix} a & b & d \\ b & c & e \\ d & e & 1 \end{pmatrix},\ \delta = a\,c - b^2,\ S = a + c. \quad (4)$$

Now, ellipse's center coordinates are

$$x_c = (b\,e - c\,d)/\delta,\ \ y_c = (b\,d - a\,e)/\delta; \quad (5)$$

diameters of the ellipse are

$$D_{1,2} = 2\,\left[-\triangle/(\delta\,u_{1,2})\right]^{1/2}, \quad (6)$$

where $u_{1,2}$ are solutions of the quadratic $u^2 - S\,u + \delta = 0$.

Finally, the angular coefficient of the X-diameter of the ellipse is

$$k = \left[c - a + \sqrt{(c-a)^2 + 4\,b^2}\right]/(2\,b). \quad (7)$$

Example of resulting fitting ellipse is shown at the right panel of Fig. 2, see also Fig. 4.

With this we finish the description of the STB procedure and pass to describe EAFEL experiments, STB treatment, EGUN simulation and comparison of the results.

EXPERIMENT AND SIMULATIONS

We consider the case when the first focusing coil current is of the *negative* sign (according to agreement in EAFEL group), that is C1 coil rotates the image counter-clockwise. Other coil current signs are positive or negative with respect to C1 current. In the case considered here (and usually used in EAFEL), the current signature is -/+/+/+ that is only I1 is negative.

Experimental data

In Fig. 3, four spots at S0 screen are shown for values of C4 coil $I4 = .8,\ 2.5,\ 3,$ and 4 Amp with three other focusing coil currents fixed as $I1 = -7.25$ A, $I2 = 3.75$ A, and $I3 = 3.6$ A. Also shown are the elliptical approximations of spots found by STB procedure.

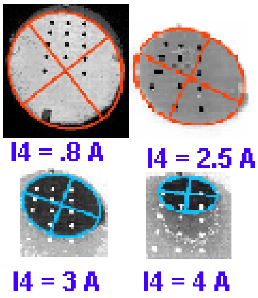

Figure 3: (Color) Spots at S0 vs. current of C4. Note that at the screen surface, the distance between neighbor holes are 10 mm in both x- and y-directions. Screen S0 is rotated around horizontal (X) axis by 45 deg, so that at images, the vertical distances between holes are (or rather should be) $1/\sqrt{2} \approx 7.1$ mm. Note also a rather strong distortion of images apparent from the holes misalignment. First, in general, the camera is too close to the screen, then it seems that the camera is closer to the upper side of the screen than to the lower side of the screen, and again it seems that the orientation angle is not exactly 45 deg. Also shown are the elliptical *spot* approximations (and reference holes!) as defined by using STB. The *beam* ellipses are shown in Fig. 4

STB treatment results

The most important is of course to present the *beam* near the given diagnostic screen as ellipse, and this is exactly what was done by STB procedure. Results of the STB procedure treatment are presented in Fig. 4, where four beams at S0 screen are shown for the values of C4 coil current $I4 = .8,\ 2.5,\ 3,$ and 4 Amp with three other coil currents fixed as $I1 = -7.25$ A, $I2 = 3.75$ A, and $I3 = 3.6$ A. With an increasing current, the elliptical beam at S0 rotates counter-clockwise, its center moving almost vertically upward, and both diameters decreasing.

Also we mention that all beams are (curiously enough) touching each other in one point.

The analysis of such a behavior of the beam with changing coil currents will be the subject of another paper.

Here we present only numerical results. For C4 coil currents $I4 = .8, 2.5, 3,$ and 4 Amp, the coordinates of beam centers, in mm, are $\{X_c, Y_c\} = \{-0.4, -11.1\}$, $\{5.1, 0.3\}$, $\{4.6, 4.9\}$, and $\{2.6, 13.2\}$, respectively.

Apparently the beam, at least near the S0 screen position, is off-axis, non-circular and moves at some angles (in XZ- and YZ-planes) in respect to Z-axis.

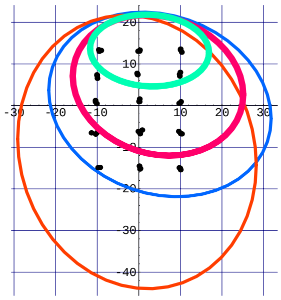

Figure 4: (Color) Beams at S0 screen position (as found by using STB procedure) for four values of current of C4 coil: .8 A (the light-red largest ellipse), 2.5 A (the dark-blue second-largest ellipse), 3 A (the dark-red second-smallest ellipse), and 4 A (the light-green smallest ellipse). With increasing current, the elliptical beam at S0 rotates counter-clockwise, its center moving almost vertically upward, and both diameters decreasing.

EGUN simulations

We used the EGUN code [3] to calculate the beam projectiles from the electron gun (of Pierce type) up to S0 screen. We present here only the current density distribution across the (circular) beam cross-section, which allow to estimate the beam dimension.

In Fig. 5, four beams at S0 screen position (found by using EGUN code) are shown for values of C4 coil $I1 = .8, 2.5, 3,$ and 4 Amp with three other coil currents fixed as $I1 = -7.25$ A, $I2 = 3.75$ A, and $I3 = 3.6$ A.

Experiment/simulation comparison

In Fig. 6, the red lines show the upper and lower diameters (obtained by using STB) of beams at S0 screen position, for values of C4 coil $I1 = .8, 2.5, 3,$ and 4 Amp, with three other coil currents fixed as $I1 = -7.25$ A, $I2 = 3.75$ A, and $I3 = 3.6$ A. The "simulated" (by EGUN) values (blue dash line) are close to the lower diameters of "experimental" beam ellipses (obtained by STB) for smaller I4 and to the upper "experimental" diameters for larger I4. Note that EGUN code (at least in its standard operational mode) can treat only axially symmetric circular

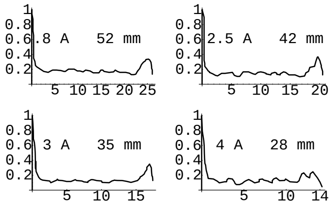

Figure 5: Current density distribution (in arbitrary units) in the cross-section of the axially-symmetric circular beam at S0 screen position, as calculated by using EGUN, for various values of C4 coil current. Value of C4 current, in Amp, and assumed beam diameter, in mm, are shown at each panel. Note a *spurious* increase of current density at the edge of beam. This "hump" in the beam edge current distribution is (unwilling) characteristic of EGUN code.

beams (with a due account of space-charge effects), while in reality, beam at SP and S0 positions are off-axis, non-circular and even not-parallel to Z-axis. This precludes the rigorous comparison of simulations and experiments. Still we mention a very good agreement between the "simulated" circular-beam diameters and the lower/upper diameters of the "experimental" beam ellipses. The relative variance in the beam diameter is about 10% for I4 = .8 Amp and is much less for the larger currents of C4 coil.

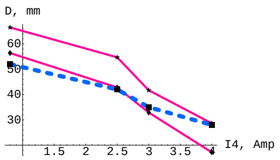

Figure 6: (Color) Beam dimensions at S0 position vs C4 coil current. Red curves: "experimental" (as obtained by using STB) minor and major diameters of elliptic beam. Blue dash line: diameter of the circular beam simulated by EGUN. Ordinates: diameters in mm; abscissas: current of C4 coil in Amp.

ACKNOWLEDGEMENTS

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