Dynamical analysis of chaos generated on a Storage Ring Free Electron Laser

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Detuning : difference between the pass frequency of the electrons in the optical klystron and the frequency of back and return of the light pulse in the optical cavity

Detuning curve

'cw' regime

Pulsed regime

Macro-temporal structures: 'cw' or pulsed at the resonance frequency
FEL sensibility to external perturbations

- **Laser rise time (10-30 µs)**
  Prevent the laser to start

- **Coherent Synchrotron oscillations (35-70 µs)**:
  Prevent the laser to stay stable at the perfect tuning

- **Laser resonant frequency (2-5 ms)**:
  Variation of the laser gain, chaotic dynamics
  Non linear gain ($g$) variation through a detuning ($\delta$) modulation (RF cavity frequency)

  $$g(\tau) = g_0 \exp\left(-\frac{(\tau+\delta)^2}{2\sigma^2}\right)$$

  and

  $$\delta(t) = a \sin (2\pi ft)$$
Time structures

Modulation of the detuning

Periodic regime of the laser intensity

Chaotic regime of the laser intensity

Intensity variation and rise up of the frequency spectrum
Maximum Lyapunov exponent

For close initial conditions, small perturbation of the difference of intensity of nearby trajectories

\[ I(t_1) \sim I(t_2) \]
\[ \Delta_0 \sim I(t_1) - I(t_2) \]
\[ \Delta_t \sim I(t_1 + t) - I(t_2 + t) \]

Sensibility to initial conditions: exponential growth \( \Delta_t = \Delta_0 \exp(\lambda t) \)

Slope of the fit: \( \lambda = 3.4 / \text{ms} \)

\( \lambda > 0 \): Deterministic chaos
Phase space portrait: periodic regime

Time serie

I(t), dI/dt, d²I/dt²

Spectrum

Limit cycle regime: same trajectory in the phase space

Experimental phase-space built from the time serie: I(t), dI/dt, d²I/dt²
Phase space portrait: chaotic regime

**Time serie**

Chaotic regime

**Spectrum**

Phase space portrait built from 2500 points

Phase space portrait built from 50000 points

Different trajectories in the phase space
Surface in the 3D space where the trajectories do not cross
Fixed frequency, increased amplitude

**Time serie**

**Spectrum**

**Phase space portrait**

*Periodic 1T*

- $a = 20 \text{ Hz}$

*Chaotic a = 28 Hz*

*Periodic 3T*

- $a = 60 \text{ Hz}$

*Chaotic a = 99 Hz*

*Periodic 2T*

- $a = 100 \text{ Hz}$

*Windows of chaos between periodic regimes*
Numerical model reproducing the longitudinal dynamics pass per pass (LAS)

Evolution of the normalized energy spread

\[
\Sigma_n = \Sigma_{n-1} - \frac{2T_0}{\tau_s} \int y_{n-1}(\tau) d\tau
\]

Evolution of the normalized laser intensity distribution

\[
y_n(\tau) = (1-L) y_{n-1}(\tau) \left[ 1 + G_0 \left( \frac{L}{G_0} \right) e^{\frac{(\tau+\delta)^2}{2\sigma_\tau^2}} \right] + \Sigma_n \left[ 1 + G_0 \left( \frac{L}{G_0} \right) e^{\frac{-(\tau-\delta)^2}{2\sigma_\tau^2}} \right] + i_s \left[ 1 + G_0 \left( \frac{L}{G_0} \right) e^{\frac{-(\tau+\delta)^2}{2\sigma_\tau^2}} \right]
\]

Normalized energy spread

\[
\Sigma_n = \frac{\sigma_{\gamma n}^2 - \sigma_{\gamma O}^2}{\sigma_{\gamma e}^2 - \sigma_{\gamma O}^2}
\]

\(\sigma_{\gamma n}\): energy spread at the pass \(n\), \(\sigma_{\gamma O}\): initial, \(\sigma_{\gamma e}\): at the laser equilibrium, \(n\): pass, \(T_0\): pass frequency of the electrons in the optical klystron, \(\tau_s\): damping time, \(\tau\): longitudinal coordinate, \(L\): optical cavity losses, \(G_0\): small signal gain, \(\delta\): the detuning, \(\sigma_\tau\): rms width of the electrons bunch.

Detuning curve

Experimental

Simulated
Fixed frequency, increased amplitude

**Time serie**

<table>
<thead>
<tr>
<th>Periodic</th>
<th>Chaotic</th>
<th>Periodic</th>
<th>Chaotic</th>
<th>Periodic</th>
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</thead>
<tbody>
<tr>
<td>1T</td>
<td>3T</td>
<td>2T</td>
<td></td>
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<tr>
<td>(a = 20) Hz</td>
<td>(a = 30) Hz</td>
<td>(a = 90) Hz</td>
<td>(a = 140) Hz</td>
<td>(a = 180) Hz</td>
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**Spectrum**

**Phase space portrait**

Experimental sequence reproduced numerically
Diagramme of the sequences

Periodic windows between **chaotic windows** for all the frequency of the modulation

Different frequency of the period regimes depending on the frequency: the oscillations tend to be **pulsed near the resonance frequency**