6th Microbunching Instability Workshop
Trieste, Italy / 6 - 8 October 2014

Elettra Sincrotrone Trieste

Controlling CSR-induced emittance growth in DBAs

-by using "2D-point kick analysis" method

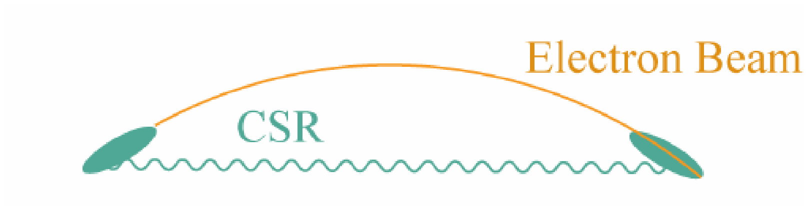
<u>Yi Jiao</u>, Xiaohao Cui, Xiyang Huang, Gang Xu

Institute of High Energy Physics, Beijing

2014-10-06, Trieste, Italy

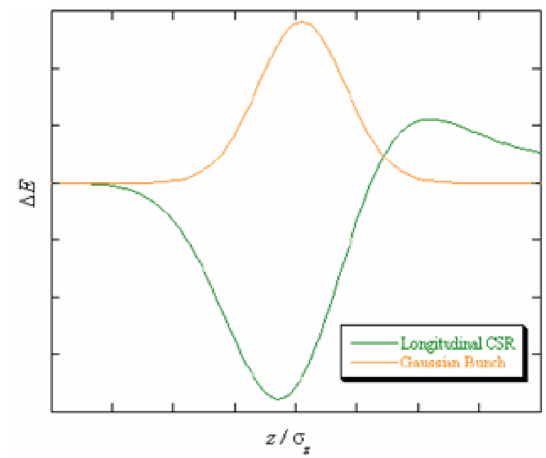
Transverse emittance dilution due to CSR

Courtesy of R. Hajima

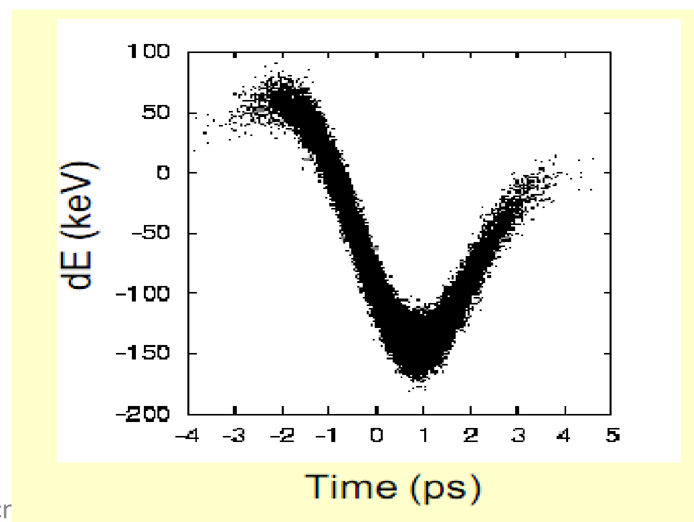


(1) CSR emission from the bunch tail catches up with the bunch head

(2) Energy change depending longitudinal position

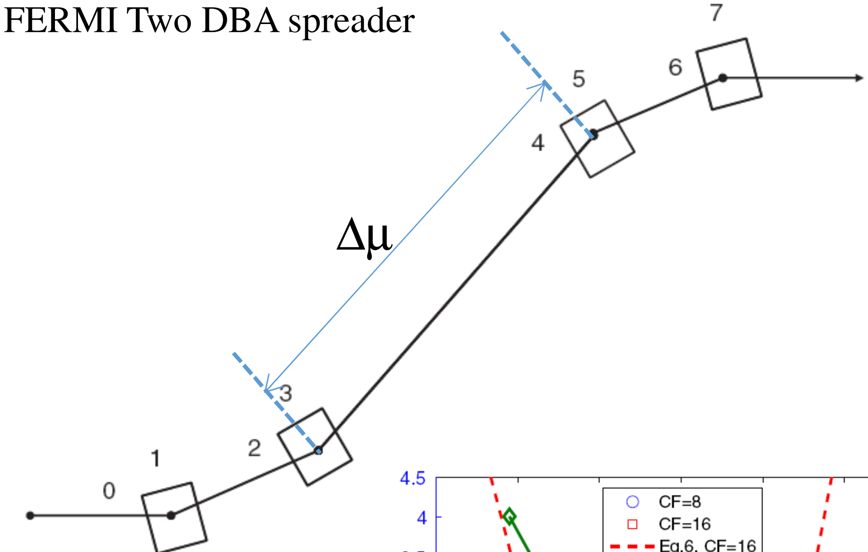


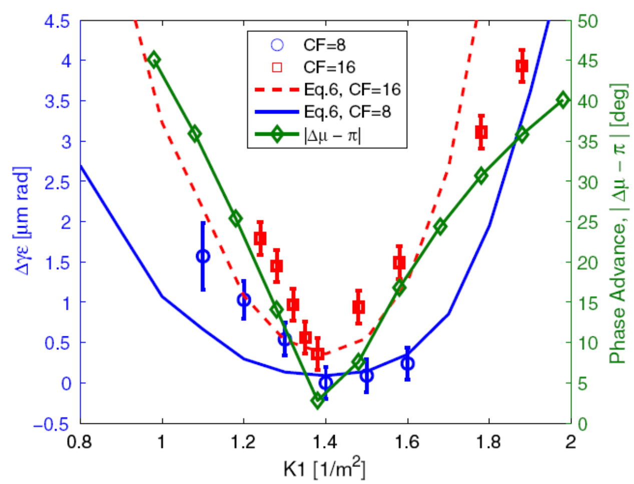
(3) Displacement of bunch slices



Cancellation of CSR kicks with optics balance

FERMI Two DBA spreader





Minimum $\Delta\gamma\varepsilon$ when $\Delta\mu = \pi$.

At "0" point:

$$x_0 = 0, \quad x_0' = 0,$$

At "1" point, the CSR kick:

$$x_{k,1} = \eta\delta, \quad x_{k,1}' = \eta'\delta,$$

At "2" point, the CSR kick:

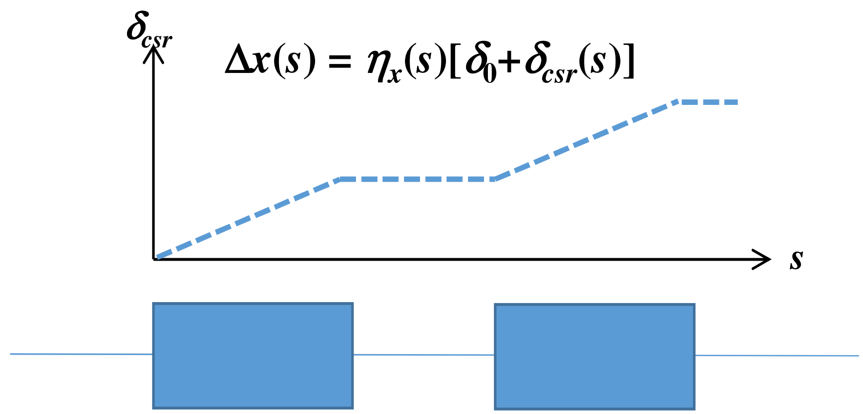
$$x_{k,2} = \eta\delta, \quad x_{k,2}' = -\eta'\delta,$$

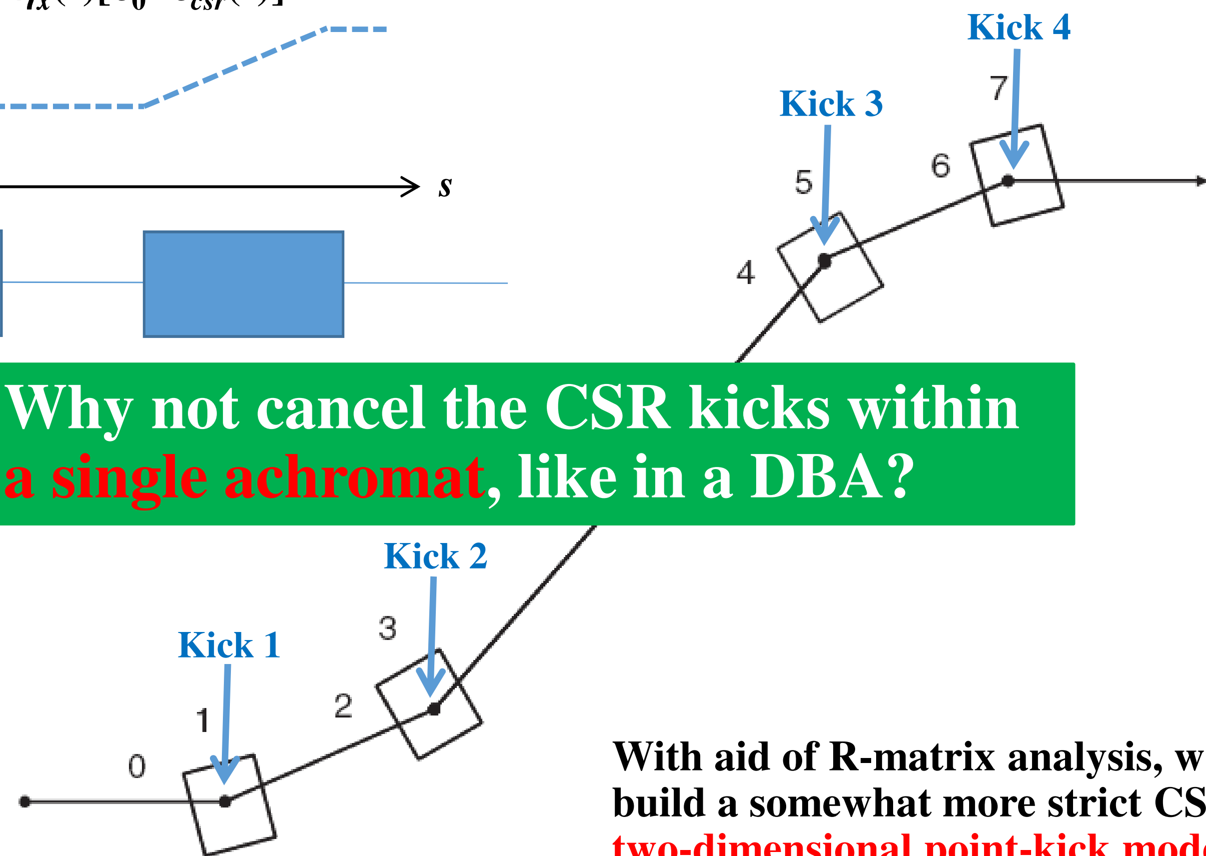
...

$$\Delta\gamma\varepsilon = \gamma\varepsilon\left[\sqrt{1 + \frac{H_1\sigma_{\delta,\mathrm{CSR}}^2}{\varepsilon}X_{17}} - 1\right],$$

S. Di Mitri, M. Cornacchia, and S. Spampinati, PRL 110, 014801, 2013.

Inspired by the CSR kick approximation & C-S formulation…





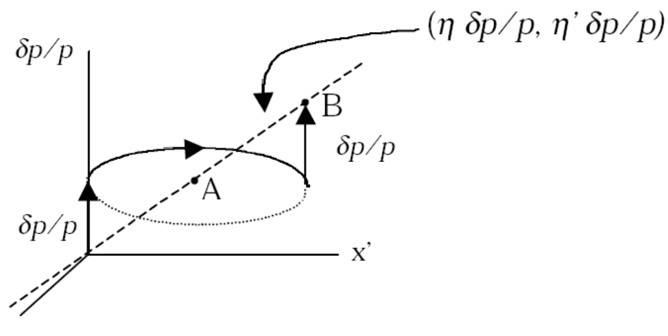
Why not cancel the CSR kicks within a single achromat, like in a DBA?

With aid of R-matrix analysis, we build a somewhat more strict CSR two-dimensional point-kick model.

A selection of CSR-study papers, not for all…

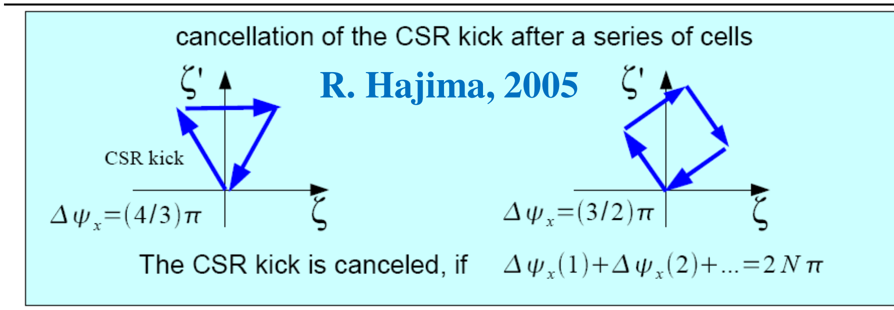
- T. Nakazato, M. Oyamada, N. Niimura, S. Urasawa, O. Konno, A. Kagaya, R. Kato, T. Kamiyama, Y. Torizuka, T. Nanba, Y. Kondo, Y. Shibata, K. Ishi, T. Ohsaka, and M. Ikezawa,, Phys. Rev. Lett. 63, 2433 (1989).
- Ya.S. Derbenev, J. Rossbach, E.L. Saldin, and V.D. Shiltsev, Deutsches Elektronen-Synchrotron Report No. TESLA-FEL 95-05, 1995.
- E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, Nucl. Instrum. Methods Phys. Res., Sect. A 398, 373 (1997).
- Ya.S. Derbenev and V.D. Shiltsev, Stanford Linear Accelerator Center Report No. SLAC-PUB-7181, 1996.
- P. Emma and R. Brinkmann, Stanford Linear Accelerator Center Report No. SLAC-PUB-7554, 1997.
- M. Dohlus and T. Limberg, Nucl. Instrum. Methods Phys. Res., Sect. A 393, 494 (1997).
- D. Douglas, Thomas Jefferson, National Accelerator Laboratory Technical Note, JLAB-TN-98-012 (1998)
- H. Braun, F. Chautard, R. Corsini, T.O. Raubenheimer, and P. Tenenbaum, Phys. Rev. Lett. 84, 658 (2000).
- M. Borland, Phys. Rev. ST Accel. Beams 4, 070701 (2001).
- R. Li and Ya. S. Derbenev, Thomas Jefferson National Accelerator Facility Report No. JLAB-TN-02-054, 2002.
- S. Heifets, G. Stupakov, and S. Krinsky, Phys. Rev. ST Accel. Beams 5, 064401 (2002).
- Z. Huang and K.J. Kim, Phys. Rev. ST Accel. Beams 5, 074401 (2002).
- R. Hajima, Nucl. Instrum. Methods Phys. Res., Sect. A 528, 335 (2004).
- G. Bassi, T. Agoh, M. Dohlus, L. Giannessi, R. Hajima, A. Kabel, T. Limberg, and M. Quattromini, Nucl. Instrum. Methods Phys. Res., Sect. A 557, 189 (2006).
- M. Shimada, M. Okazaki, K. Harada, O. Tsukuba, in Proceeding of ERL07, Daresbury, UK (2007), p. 108.
- D. Sagan, G. Hoffstaetter, C. Mayes, and U. Sae-Ueng, Phys. Rev. ST Accel. Beams 12, 040703 (2009).
- C. Mayes and G. Hoffstaetter, Phys. Rev. ST Accel. Beams 12, 024401 (2009).
- V. Yakimenko, M. Fedurin, V. Litvinenko, A. Fedotov, D. Kayran, and P. Muggli, Phys. Rev. Lett. 109, 164802 (2012).
- C. Mitchell, J. Qiang, and P. Emma, Phys. Rev. ST Accel. Beams 16, 060703 (2013).
- S. Di Mitri, M. Cornacchia, and S. Spampinatic, Phys. Rev. Lett. 110, 014801 (2013).

Suppress CSR by optics balance or optics symmetry

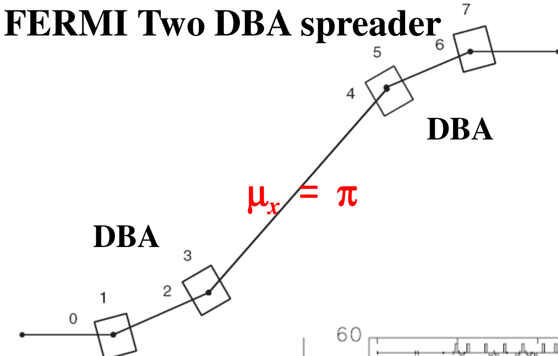


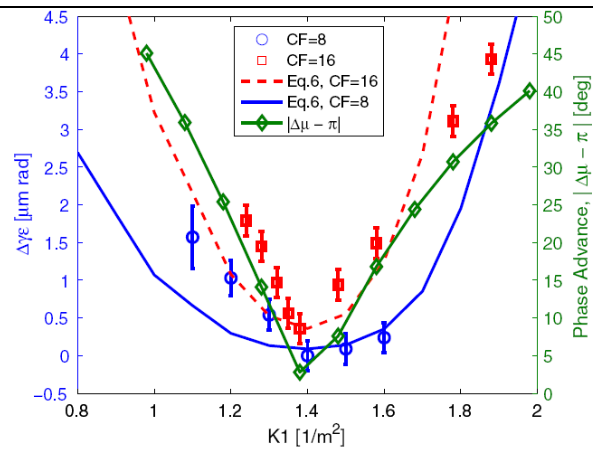
Periodic transport, with half integer phase advance between two identical periods. Electrons experience the same CSR kicks at two periods. With –I transportation, the CSR kicks are cancelled.

D. Douglas, JLAB-TN-98-012, 1998



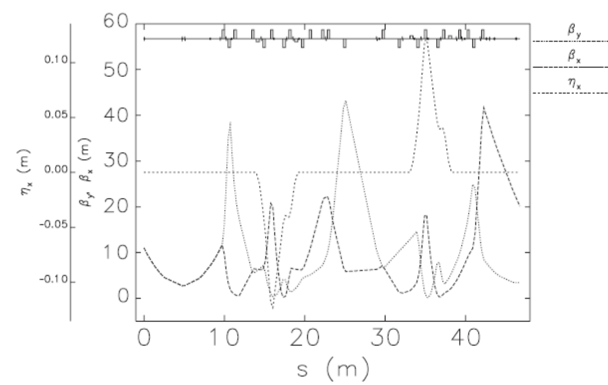
FERMI Two DBA spreader





When the phase advance between two DBAs is π, the CSR induced emittance growth can be minimized.

S. Di Mitri, M. Cornacchia, and S. Spampinati, PRL 110, 014801, 2013



Linearization of CSR-induced energy spread, for linear analysis

$$\Delta E_{rms} \cong 0.22 \frac{eQL_b}{4\pi\varepsilon_0 \rho^{2/3} \sigma_z^{4/3}}$$

Bunch charge (Q); Bending path (L_b); Bunch length (σ_z); Bending radius (ρ)

If we assume that:

1. **The bunch length σ_z does not change a lot along the transport line**
2. **The transient CSR effect is not large**
3. **Bending angles of the dipoles are not very large, $< 10^\circ$**

The CSR induced energy spread can be linearized

$$\Delta E(csr)/E_0 \cong \kappa\theta$$

$\kappa = f(Q,\sigma_z, \rho)$, unit: m^{-1}

By R. Hajima

$$\Delta E(csr)/E_0 \cong k\rho^{1/3}\theta$$

$k = f(Q,\sigma_z)$, unit: $m^{-1/3}$

In 2D point-kick analysis

Linear matrix analysis of the CSR effect

Courtesy of R. Hajima

We assume
- all the dipoles have the same bending radius,
- the bunch does not change its longitudinal profile,
- the transient CSR effect is not large.

→ constant CSR wake regime, valid for GeV ERLs

we can attribute κ to each electron

electron's motion in a bending plane

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}\left(\delta_0 + \underbrace{\delta_{CSR} + \kappa[s - s_0]}_{\text{for CSR}}\right)$$

$$\vec{x}(s) = (x, x', \delta_0, \kappa L_b, \kappa)^T$$

5x5 R-matrix for a sector bending magnet

$$R_{bend} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) \\ -\rho^{-1}\sin\theta & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

extension of the conventional 3x3 R-matrix
R. Hajima, JJAP 42, L974 (2003).

x deviation from the reference path
ρ curvature radius of the bending
δ_0 initial momentum error
s coordinate along the path
κ normalized CSR wake potential
δ_{CSR} momentum error by CSR in the upstream path

Following the momentum dispersion function, define the CSR wake dispersion function

momentum dispersion function

(η, η')

$$\begin{pmatrix} \eta_x(s_1) \\ \eta'_x(s_1) \\ 1 \\ 0 \\ 0 \end{pmatrix} = R_{0\to1} \begin{pmatrix} \eta_x(s_0) \\ \eta'_x(s_0) \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

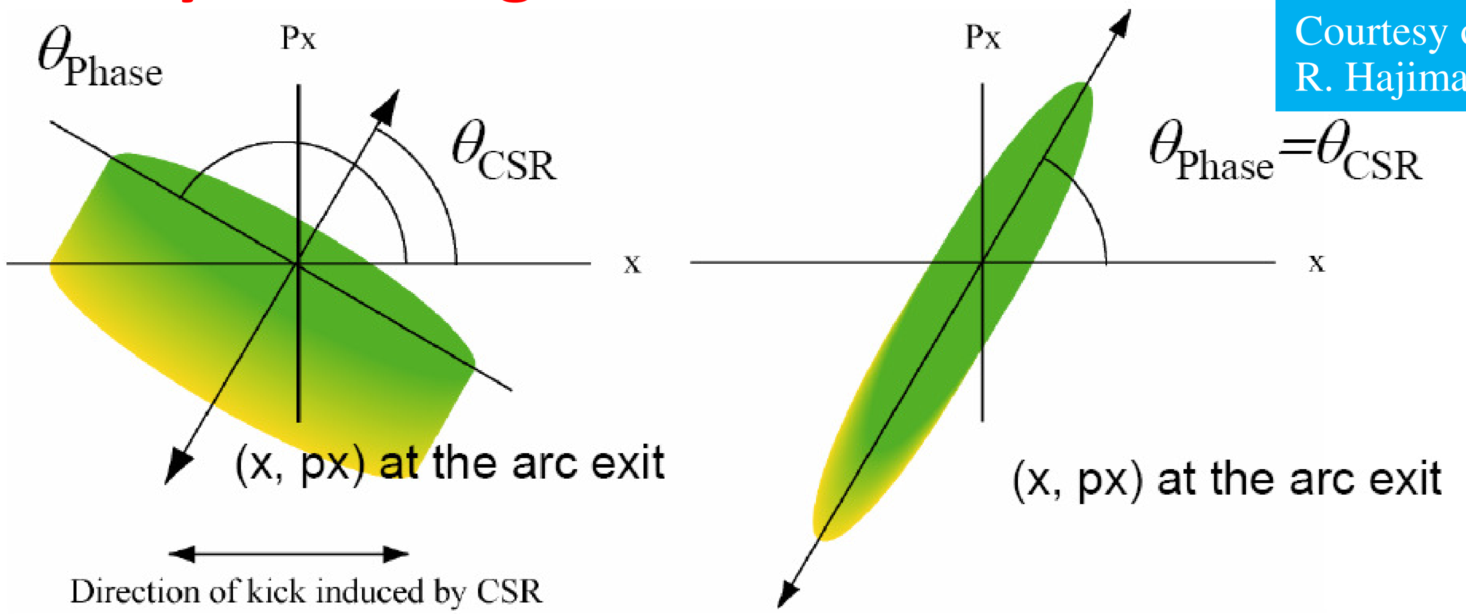
CSR wake dispersion function

(ζ, ζ')

$$\begin{pmatrix} \zeta_x(s_1) \\ \zeta'_x(s_1) \\ 0 \\ L_b(s_1) \\ 1 \end{pmatrix} = R_{0\to1} \begin{pmatrix} \zeta_x(s_0) \\ \zeta'_x(s_0) \\ 0 \\ L_b(s_0) \\ 1 \end{pmatrix}$$

Suppression by matching net CSR kick to beam envelope

Courtesy of
R. Hajima



Large emittance growth **Minimized emittance growth**

The emittance growth is minimized when θ_{Phase} coincides with θ_{CSR} (direction of CSR kick).

R. Hajima, Nuclear instruments and Methods in Physics Research A 528 (2004) 335-339

$$\tan 2\theta_{Phase} = 2\alpha/(\gamma-\beta) \qquad \tan\theta_{CSR} = \sin\phi/\rho(1-\cos\phi)$$

CSR effect in dipole described with a 2D point kick

R-matrix analysis predicts:

$$\Delta X_{f,RM} = \begin{pmatrix} D \\ D' \end{pmatrix} \delta_i + \begin{pmatrix} \zeta \\ \zeta' \end{pmatrix} k,$$

with $D = \rho(1-\cos\theta)$, $D' = \sin\theta$,
momentum dispersion (x-δ correlation),
and
$\zeta = \rho^{4/3}(\theta - \sin\theta)$, $\zeta' = \rho^{1/3}(1-\cos\theta)$,
"CSR dispersion" (x-k correlation).

CSR point-kick model:
1, It occurs at the dipole center
2, The kick is **in a similar form**,

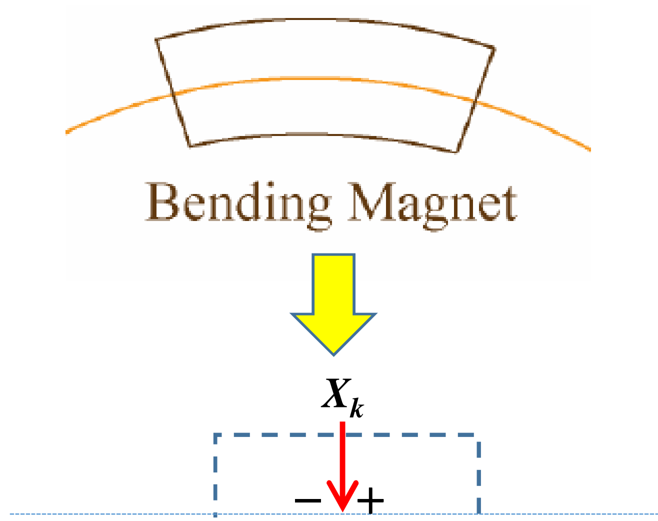
$$X_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix} = \begin{pmatrix} D_k \\ D'_k \end{pmatrix} \delta_i + \begin{pmatrix} \zeta_k \\ \zeta'_k \end{pmatrix} k,$$

and **predicts the same coordinate deviation** at the end of the dipole,

$$\Delta X_f = M_d(\theta/2) X_k.$$

Betatron transfer matrix:

$$M_d = \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\sin\theta/\rho & \cos\theta \end{pmatrix}$$

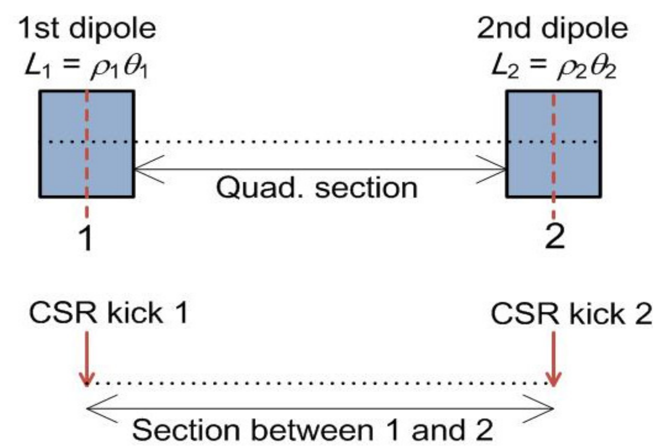


$$\mathbf{X_+ = X_- + X_k}$$
$$\boldsymbol{\delta_+ = \delta_- + k\rho^{1/3}\theta}$$

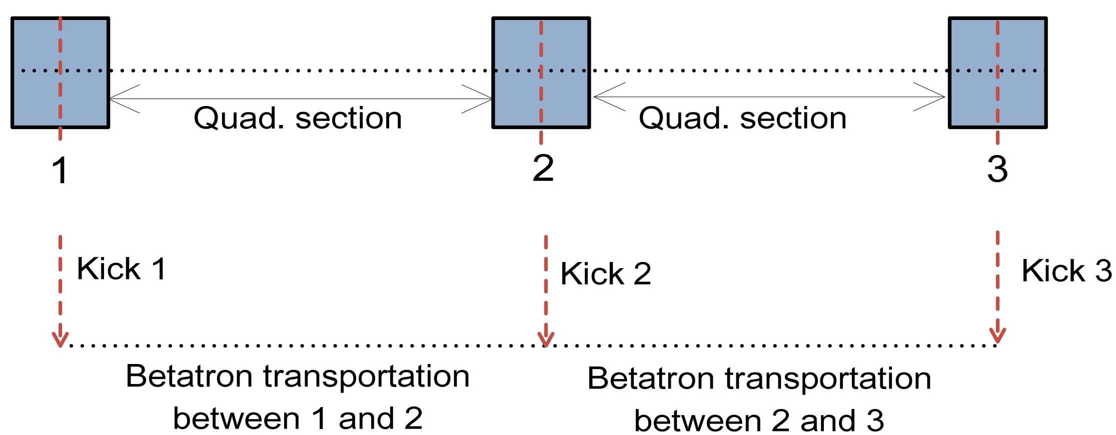
$$X_k = \begin{pmatrix} 0 \\ 2\sin(\theta/2) \end{pmatrix} \delta_i + \begin{pmatrix} \rho^{4/3}[\theta\cos(\theta/2) - 2\sin(\theta/2)] \\ \sin(\theta/2)\rho^{1/3}\theta \end{pmatrix} k.$$

2D point-kick analysis for achromats

- For an *n*-dipole achromat, it needs only to analyze the horizontal betatron motion with *n*-point kicks, explicit formulation;
- The beam line between adjacent dipole centers is treated as a whole, so the obtained "zero CSR-kick" solutions predict general requirements on optics design, generic CSR-kick cancellation conditions.



The bending radii and angles can be different.



2D point-kick analysis for a two-dipole achromat

- The transfer matrix of the quad. section between dipole centers is described in a general form:

$$M_{c2c} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The net CSR kick:

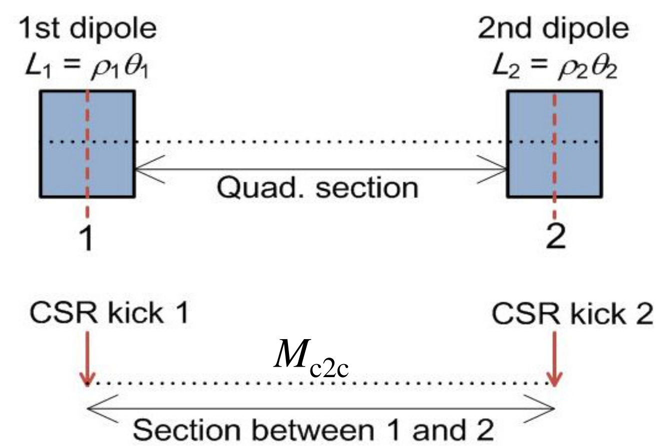
$$\Delta X = X_{k2} + M_{c2c} X_{k1}$$

- The achromatic condition [$\Delta X(\delta) = 0$]:

$$m_{11} = -S_1 / S_2$$
$$m_{12} = 0$$
$$m_{22} = -S_2 / S_1$$

Phase advance between dipole centers: $n\pi$ (applies to $m_{12} = 0$)

$$S_1 = \sin(\theta_1 / 2)$$
$$S_2 = \sin(\theta_2 / 2)$$



- CSR-kick cancellation in linear regime [$\Delta X(k) = 0$]:

$$L_1\theta_1^2 \cong L_2\theta_2^2$$

Automatically satisfied in DBA or dogleg with $L_1 = L_2$ and $\theta_1 = \theta_2$

$$m_{21} \cong \frac{12}{L_1}\frac{S_2}{S_1}$$

These results have been verified with R-matrix analysis in a more complex form.

CSR kick cancellation in a two-dipole achromat, DBA, dogleg

- **In a two-dipole achromat**

Arbitrary θ & ρ

$$L_1\theta_1^2 \cong L_2\theta_2^2,$$

$$M_{c2c} \cong \begin{pmatrix} -\frac{S_1}{S_2} & 0 \\ \frac{12}{L_1}\frac{S_2}{S_1} & -\frac{S_2}{S_1} \end{pmatrix}.$$

$S_1 = sin(\theta_1/2)$, $S_2 = sin(\theta_2/2)$,

θ_1: bending angle of the 1st dipole,

θ_2: bending angle of the 2nd dipole,

L_1: length of the 1st dipole,

L_2: length of the 2nd dipole,

M_{c2c}: 2-by-2 transfer matrix between two dipole centers.

- **In a DBA**

$\theta_1 = \theta_2$ & $\rho_1 = \rho_2$

$$M_{c2c} \cong \begin{pmatrix} -1 & 0 \\ \frac{12}{L_1} & -1 \end{pmatrix}.$$

- **In a dogleg**

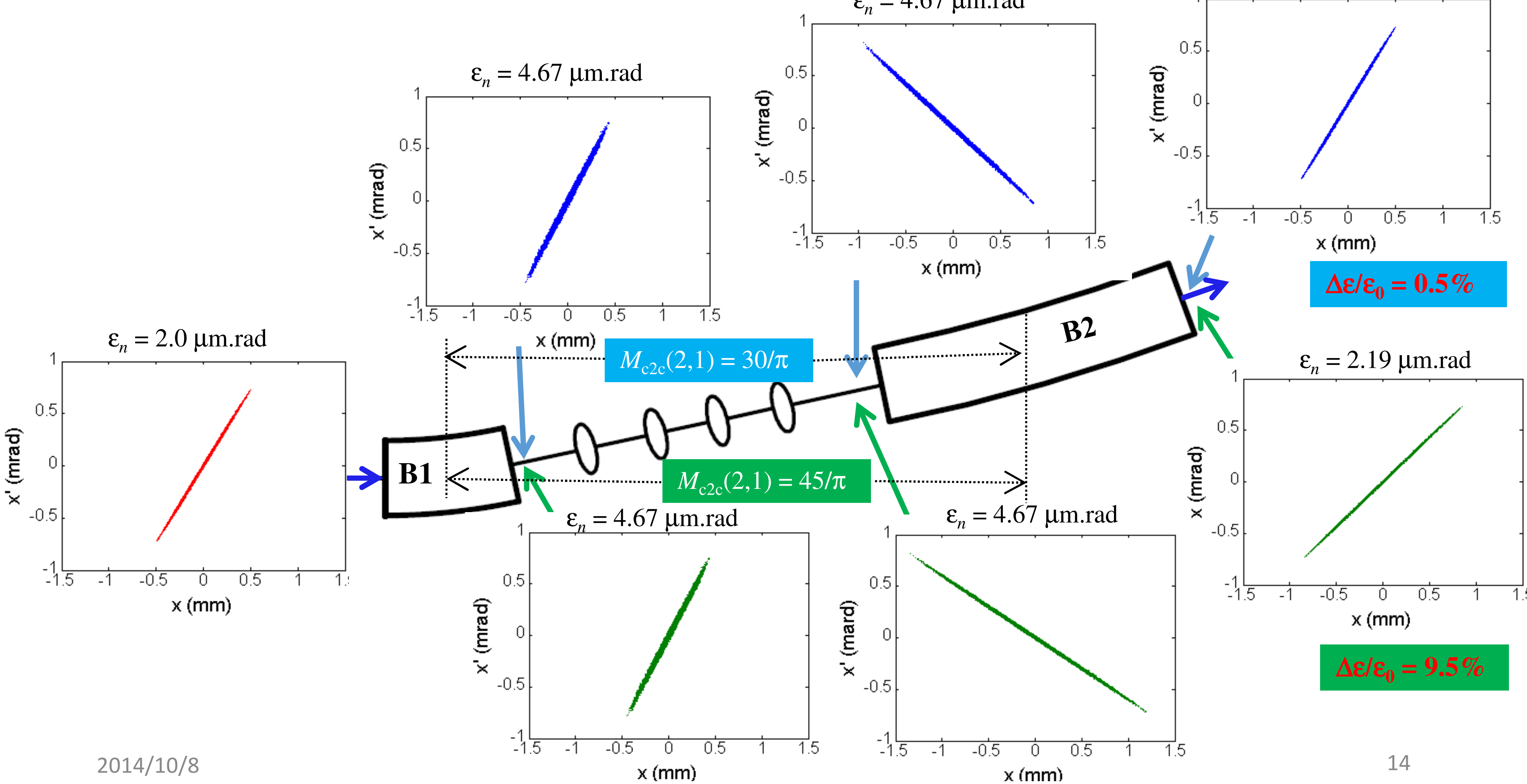
$\theta_1 = -\theta_2$ & $\rho_1 = -\rho_2$

$$M_{c2c} \cong \begin{pmatrix} 1 & 0 \\ -\frac{12}{L_1} & 1 \end{pmatrix}.$$

Design a "CSR-cancellation" two-dipole achromat

- Consider a two-dipole achromat, with $\theta_1 = 6$ deg., $\theta_2 = 4$ deg., $\rho_1 = 8$ m.

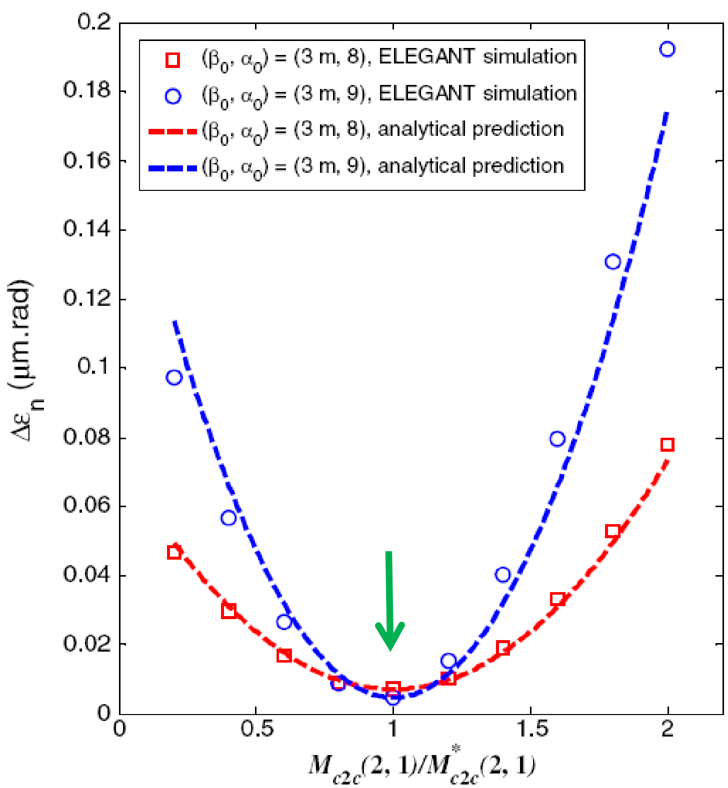
➡ $\rho_2^* \cong 27$ m, and $M^*_{c2c}(2, 1) = 30/\pi$ m

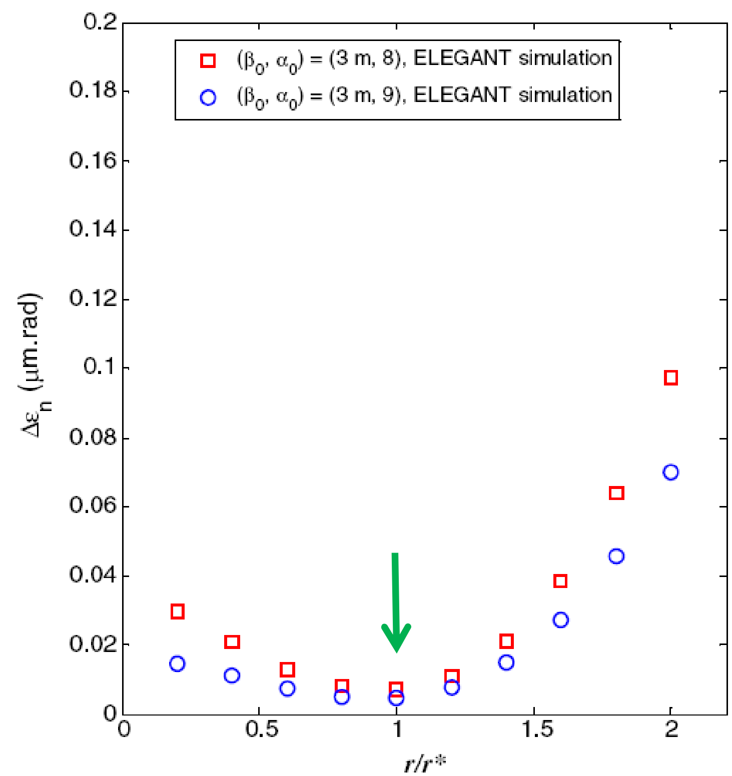


Scaling of the emittance growth due to CSR

—ELEGANT simulation

- The CSR wake in dipoles included in the tracking
  - The found conditions predict minimum emittance growth,
  - The found conditions are robust against variation of the initial beam distribution,
  - Quadratic increase of Δε as $M_{c2c}(2,1)$ moves away from the optimal value.





$$\Delta\varepsilon_n|_{r=r^*} \approx \frac{1}{2}\gamma\beta k_{rms}^2 S_1^2(\theta_1 + r^{*1/3}\theta_2)^2 \rho_1^{2/3}\beta_1[1-M_{c2c}(2,1)/M^*_{c2c}(2,1)]^2.$$

$r \equiv \rho_2/\rho_1$, and $r^* = 27/8$ in this case

Effect of 1D CSR wake when linear CSR effect is cured

—ELEGANT simulation

CSR effects	Initial normalized emittance ε_{n0} (mm.rad)	Final normalized emittance ε_{nf} (mm.rad)	Relative emittance growth $\Delta\varepsilon_n/\varepsilon_{n0}$
n.c. CSR	2	2.0030	1.5×10^{-3}
n.c. CSR + tr.CSR	2	2.0056	2.8×10^{-3}
n.c. CSR + tr.CSR +d.s.CSR	2	2.0119	5.95×10^{-3}

n.c.CSR: nonlinear components of the CSR wake in a dipole;
tr. CSR: transient CSR at the edges of the dipole;
d.s. CSR: the CSR wake in drift spaces following dipoles

Main parameters used in simulation ⟶

Parameter	Value	Unit
Bunch charge	500	pC
Beam energy	1000	MeV
Energy spread	0.05	%
Beam normalized emittance	2	μm · rad
Bunch length	30	μm
Dipole bending radius	7	m
Dipole bending angle	3	degrees

Comparison with other methods, ELEGANT simulation

- CSR-kick cancellation **vs.**

  CSR-kick matching

  in a DBA with symmetric optics

In such a case, CSR kicks cancel at the condition:

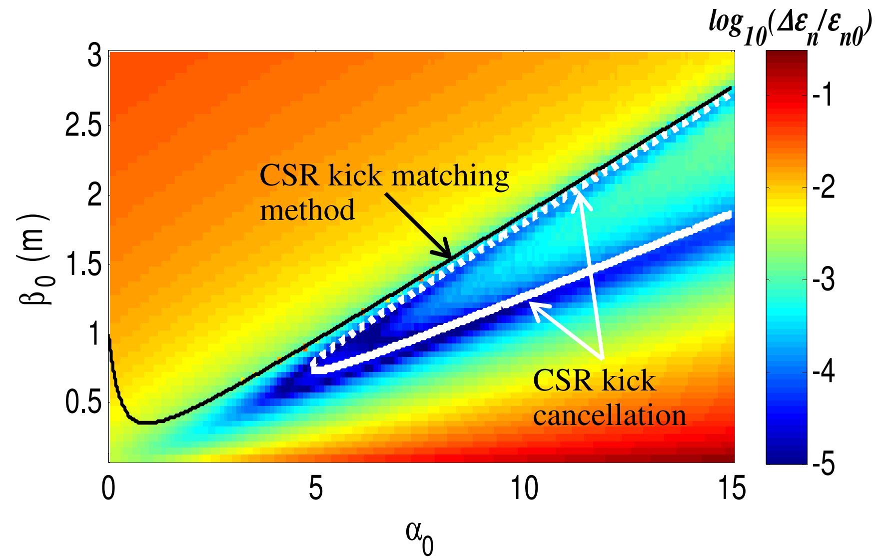
$$-\frac{2\alpha_1}{\beta_1} = m_{21} \cong -\frac{12}{L_1}$$

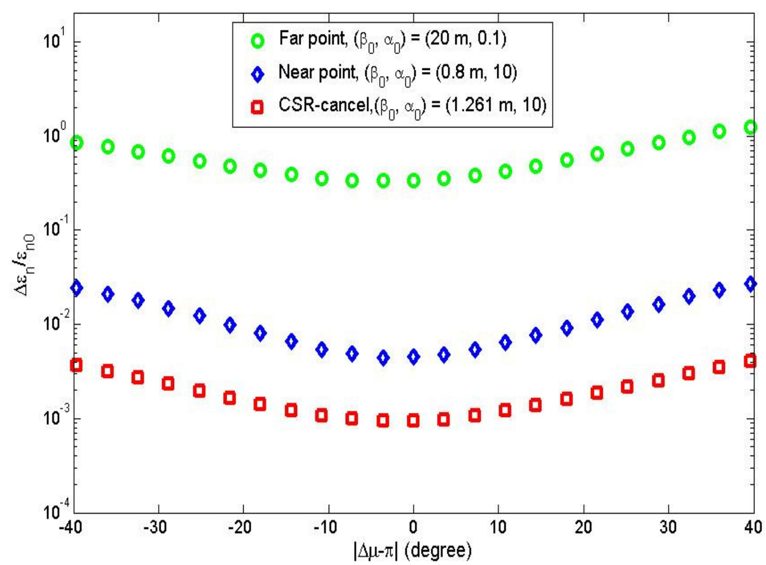
- CSR-kick cancellation **vs.**

  Optics balance (Δμ = π)

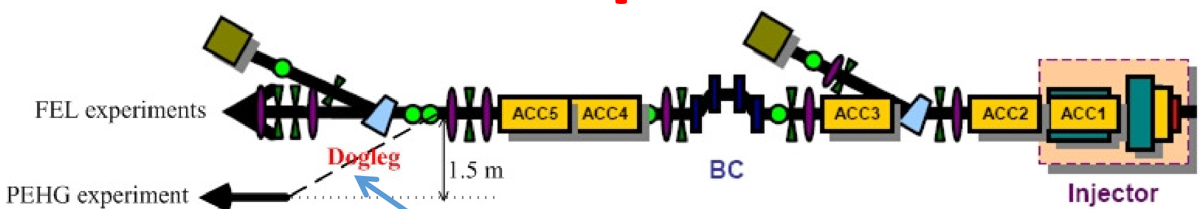
  in a FERMI spreader-like beam line

  (two identical DBAs with quads in between)



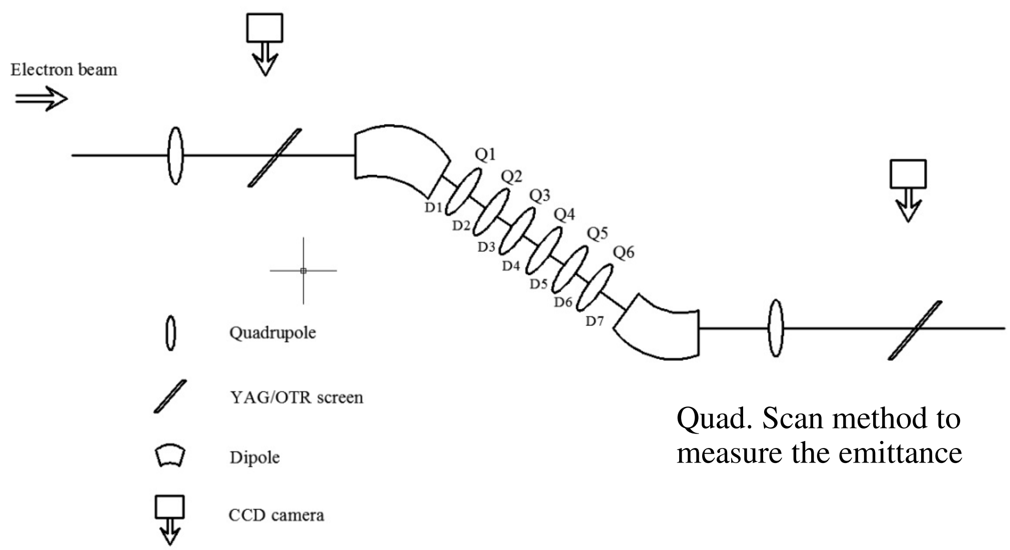


Demonstration experiment on SDUV-FEL



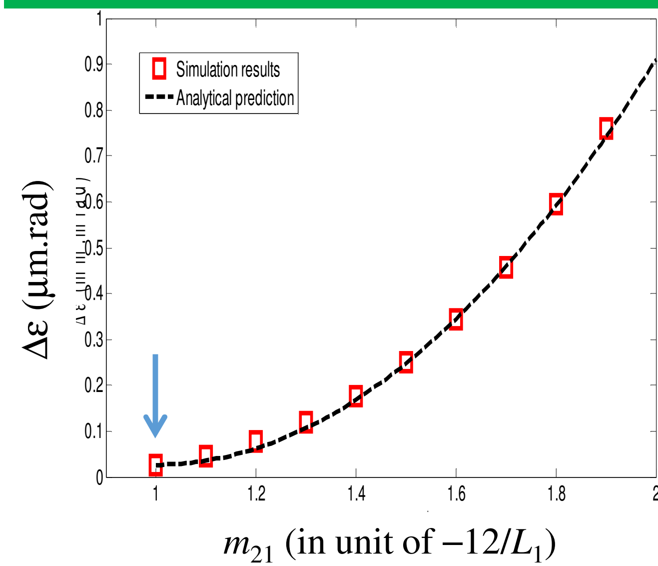
CSR suppress experiment

Experiment layout



Quad. Scan method to measure the emittance

Expected result, as $m_{21}= -12/L_1$, minimum emittance growth



A new scheme to cancel the CSR kick in DBAs (or doglegs)

1, Explicit condition:

$$M_{c2c} \cong \begin{pmatrix} -1 & 0 \\ 12/L_B & -1 \end{pmatrix}, \textbf{ or } M_{c2c} \cong \begin{pmatrix} 1 & 0 \\ -12/L_B & 1 \end{pmatrix}.$$

2, Easily applied in real machines:

Needs only tuning the quads between dipoles.

3, Robust against the variation of the phase space distribution bunch by bunch:

The CSR kick cancellation largely independent of concrete C-S parameters of the DBA.

4, Excellent suppression efficiency:

Complete cancellation of the CSR kick in linear regime.

Application of the 2D point-kick analysis to specified functional bunch compressors (where σ_z has a significant change) is in progress.

Thanks for your attention!

Backup slides

Conclusions

Consider long. CSR wake in free space, short bunch (tens of μm), low emittance

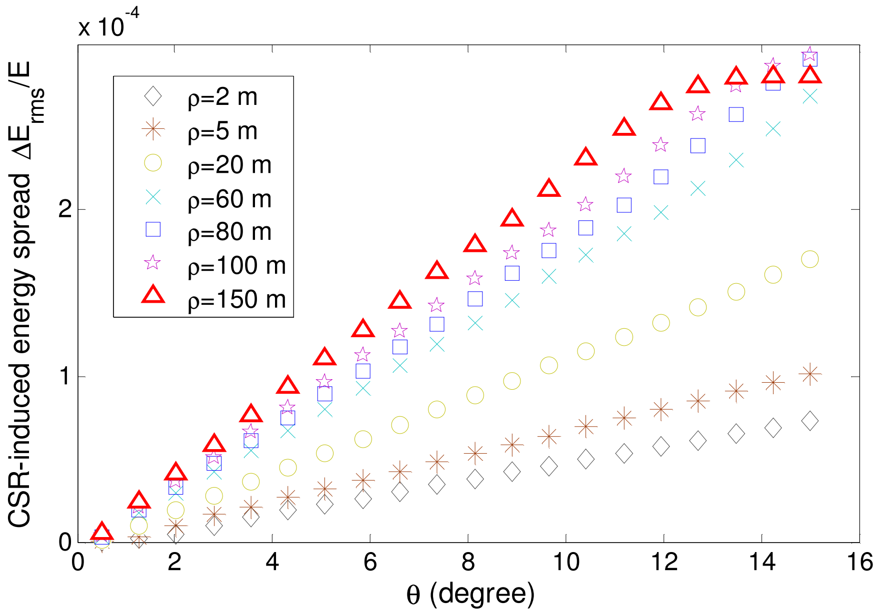
- 1, 2D point-kick analysis promises explicit formulation of the net CSR kick in achromats;
- 2, this method results in generic conditions to cure the CSR kick in linear regime and minimizes the CSR-induced geometric emittance growth;
- 3, the obtained conditions are robust against the variation of the initial beam distribution;
- 4, it suggests easily-applied CSR-suppression scheme. Most times it needs only to vary the strengths of the quadrupoles. An demonstration experiment has been suggested on SDUV-FEL in Shanghai.

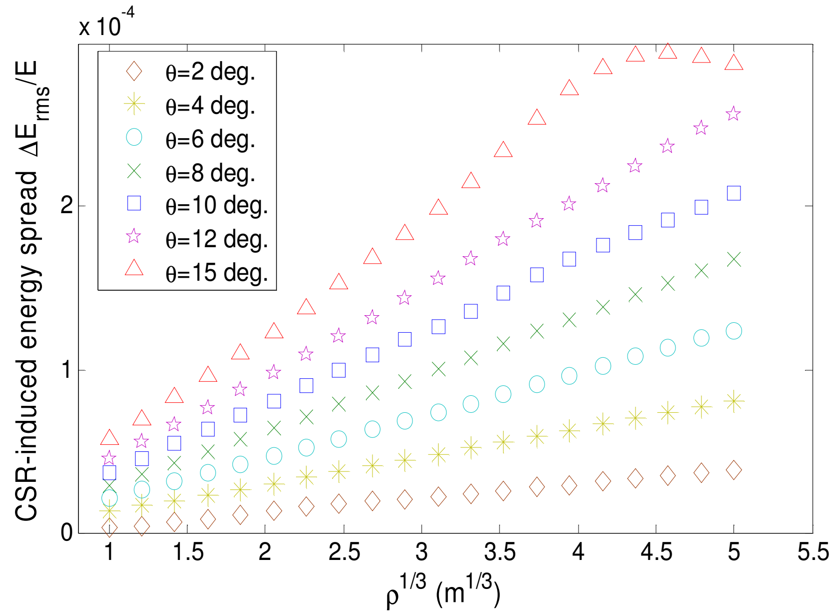
Presently the solutions are applicable to spreaders of FELs, recirculation loops of ERLs, where the bunch length does not have significant change. In near future, this method can be potentially expanded to suppress the CSR effect in specified functional bunch compressors.

Linear dependency of the energy spread vs. $\rho^{1/3}$ & θ

$$\text{If fix } \theta,\ \Delta E(csr)/E_0 \propto \rho^{1/3}$$

$$\text{If fix } \rho,\ \Delta E(csr)/E_0 \propto \theta$$





This linear relation applies well to the cases with θ from 1 to 12 degrees and ρ from 1 to 150 m.

CSR-induced orbit deviation in a bending magnet

Betatron transfer matrix :

$$M_d = \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\sin\theta/\rho & \cos\theta \end{pmatrix}$$

$\delta_i = 0$, w/o CSR effect :

$$X_f = \begin{pmatrix} x_f \\ x_f' \end{pmatrix} = MX_i = M\begin{pmatrix} x_i \\ x_i' \end{pmatrix},$$

$$x_f = m_{11}x_i + m_{12}x_i', \quad x_f' = m_{21}x_i + m_{22}x_i'.$$

$\delta_i \neq 0$, w/o CSR effect :

$$X_f = MX_i + \begin{pmatrix} D \\ D' \end{pmatrix}\delta_i,$$

$$x_f = m_{11}x_i + m_{12}x_i' + D\delta_i, \quad x_f' = m_{21}x_i + m_{22}x_i' + D'\delta_i.$$

(D, D') : momentum dispersion (x-δ correlation terms), $D = \rho(1-\cos\theta)$, $D' = \sin\theta$.

$\delta_i \neq 0$, w/ CSR effect :

$$X_f = MX_i + \begin{pmatrix} D \\ D' \end{pmatrix}\delta_i + \begin{pmatrix} \zeta \\ \zeta' \end{pmatrix}k,$$

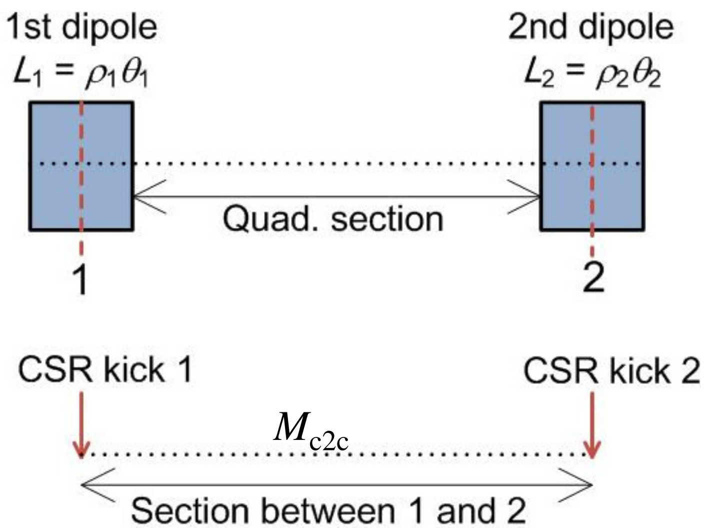
$$x_f = m_{11}x_i + m_{12}x_i' + D\delta_i + \zeta k, \quad x_f' = m_{21}x_i + m_{22}x_i' + D'\delta_i + \zeta' k.$$

(ζ, ζ') : "CSR dispersion" (x-k correlation terms), $\zeta = \rho^{4/3}(\theta - \sin\theta)$, $\zeta' = \rho^{1/3}(1-\cos\theta)$.

In addition, $\delta_f = \delta_i + k\rho^{1/3}\theta.$

R. Hajima, R-matrix analysis, NIMA 2004.

2D point-kick analysis for a two-dipole achromat



1, For simplicity, assume
$X_0 = (0, 0)^T$, $\delta = \delta_0$;

2, Right **before** the **1st** kick,
$X_{1-} = (0, 0)^T$, $\delta = \delta_0$;

3, Right **after** the **1st** kick,
$X_{1+} = X_{1-} + X_{k,1}$, $\delta = \delta_0 + k\rho_1^{1/3}\theta_1$;

4, Right **before** the **2nd** kick,
$X_{2-} = M_{c2c}X_{1+}$, $\delta = \delta_0 + k\rho_1^{1/3}\theta_1$;

5, Right **after** the **2nd** kick,
$X_{2+} = X_{2-} + X_{k,2}$, $\delta = \delta_0 + k\rho_1^{1/3}\theta_1 + k\rho_2^{1/3}\theta_2$;

Net CSR kick:

$$X_{2+} = X_{k,2} + M_{c2c}X_{k,1}$$

Final geometric emittance:

$$\varepsilon = \sqrt{(\varepsilon_0\beta_2 + x_{2+,rms}^2)(\varepsilon_0\gamma_2 + x'^2_{2+,rms}) - (\varepsilon_0\alpha_2 - x_{2+,rms}x'_{2+,rms})^2} = \sqrt{\varepsilon_0^2 + \varepsilon_0 d\varepsilon_1},$$

$$d\varepsilon_1 = \gamma_2 x_{2+,rms}^2 + 2\alpha_2 x_{2+,rms}x'_{2+,rms} + \beta_2 x'^2_{2+,rms}.$$

2D point-kick analysis for a two-dipole achromat

M_{c2c}: the betatron transfer matrix between two dipole centers

$$M_{c2c} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

Net CSR kick:

$$X_{2+} = \begin{pmatrix} 2m_{12}S_1 \\ 2(m_{22}S_1 + S_2) \end{pmatrix}\delta_0 + \begin{pmatrix} m_{12}S_1\theta_1\rho_1^{1/3} + \rho_1^{4/3}[m_{11}(C_1\theta_1 - 2S_1) + r^{4/3}(C_2\theta_2 - 2S_2)] \\ (m_{22}S_1 + 2S_2)\theta_1\rho_1^{1/3} + m_{21}(C_1\theta_1 - 2S_1)\rho_1^{4/3} + S_2\theta_2\rho_2^{1/3} \end{pmatrix} k$$

$with\ S_1 = \sin(\theta_1/2),\ C_1 = \cos(\theta_1/2),\ S_2 = \sin(\theta_2/2),\ C_1 = \cos(\theta_1/2).$

For a two-dipole achromat:

The element $\propto \delta_0$ should be zero ⇒

$$M_{c2c} = \begin{pmatrix} -S_1/S_2 & 0 \\ m_{21} & -S_2/S_1 \end{pmatrix}$$

For a two-dipole achromat, the horizontal phase advance between two dipole centers is π or 2π, only $M_{c2c}(2, 1)$ is variable.

2D point-kick analysis for a two-dipole achromat

With the achromatic condition, net CSR kick:

$$X_{2+} = \left(\frac{\rho_1^{4/3} S_1 (2S_1 - C_1\theta_1) / S_2 - \rho_2^{4/3} (2S_2 - C_2\theta_2)}{S_2(\theta_1\rho_1^{1/3} + \theta_2\rho_2^{1/3}) - m_{21}(2S_1 - C_1\theta_1)\rho_1^{4/3}} \right) k$$

$$with\ S_1 = \sin(\theta_1/2),\ C_1 = \cos(\theta_1/2),\ S_2 = \sin(\theta_2/2),\ C_1 = \cos(\theta_1/2).$$

Δε = 0 if the element ∝ *k* becomes 0 in a two-dipole achromat

Keep the first significant terms with respect to θ_1 and θ_2

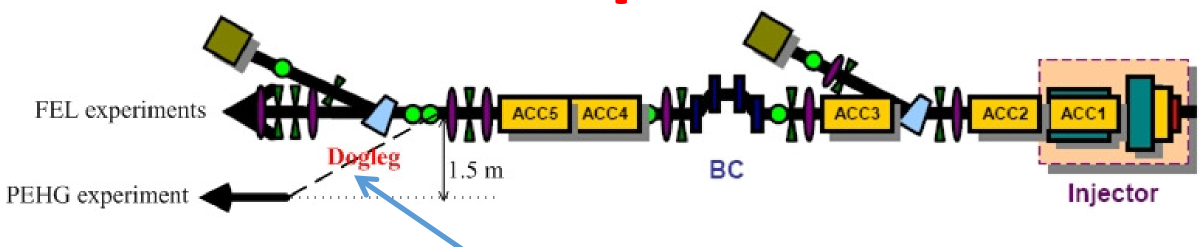
$$\left(\frac{\rho_2}{\rho_1}\right)^{4/3} = \frac{S_1(2S_1 - C_1\theta_1)}{S_2(2S_2 - C_2\theta_2)} \approx \left(\frac{\theta_1}{\theta_2}\right)^4$$

$$m_{21} = \frac{S_2}{S_1} \frac{(\theta_1\rho_1^{1/3} + \theta_2\rho_2^{1/3})}{(2 - C_1\theta_1 / S_1)\rho_1^{4/3}} \approx \frac{12}{L_1} \frac{\theta_2}{\theta_1}$$

$$L_1\theta_1^2 \cong L_2\theta_2^2$$

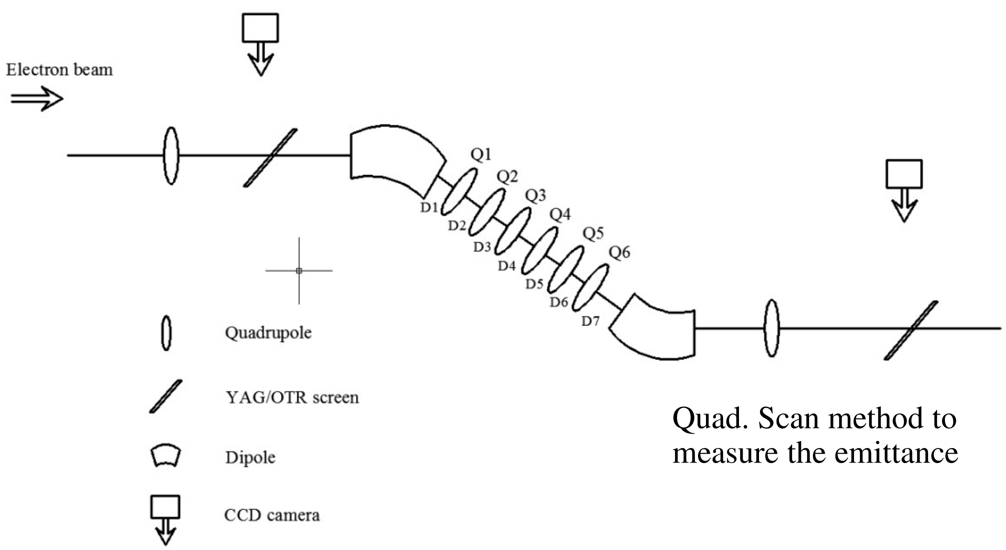
$$M_{c2c}(2,1) \cong \frac{12}{L_1} \frac{\theta_2}{\theta_1}$$

Demonstration experiment on SDUV-FEL



CSR suppress experiment

Experiment layout



Quad. Scan method to measure the emittance

Expected result, as $m_{21} = -12/L_1$, minimum emittance growth